Algorithmically on vertices that belong to all, some and no minimum dominating set in a tree

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A subset D of V_G is said to be a *dominating set* of a graph $G = (V_G, E_G)$ if each vertex in the set $V_G \setminus D$ has a neighbour in D. The (*independent*) *domination number* of G, denoted by $\gamma(G)$ (resp., by $\gamma_i(G)$), is defined to be the minimum cardinality of a (independent) dominating set D of G, and any minimum (independent) dominating set of G is referred to as a γ-set (resp., as a γ_i-set).

We propose a linear time algorithm for determining the sets of vertices that belong to all, some and no minimum dominating set of a tree, respectively, thus improving the quadratic time algorithm of Benecke and Mynhardt in 2008 [S. Benecke, C.M. Mynhardt, Trees with domination subdivision number one, *Australasian Journal of Combinatorics* 42, 201-209 (2008)].

Our result immediately implies the following corollaries: *For any tree T, the following problems are solvable in linear time and space:* (A) *The problem of verifying whether T is γ-excellent;* (B) *The problem of verifying whether* $\mathrm{sd}_\gamma(T) = 1$*;* (C) *The problem of verifying whether* $\mathrm{sd}_{\gamma_i}(T) = 1$. Recall that for a given graph parameter μ, the μ-subdivision number sd_μ is defined to be the minimum number of edges that must be subdivided to change μ, where each edge may be subdivided at most once.