Cubic vertex-transitive graphs with a symmetry-invariant 2-factor – a generalisation of the Generalised Petersen graphs

P. Šparl[1]

[1] University of Ljubljana, Ljubljana, Slovenia

We present a few results obtained during the investigations spurred by the question of Bojan Mohar to Brian Alspach from 2016 on cubic vertex-transitive graphs admitting a 2-factor which is invariant under the full automorphism group of the graph. It is not difficult to show that the graphs admitting such a 2-factor are precisely the cubic vertex-transitive but not arc-transitive graphs, but obtaining a precise description of the graphs in terms of the corresponding 2-factor and the complementing 1-factor presents an interesting challenge.

While the simplest case when the 2-factor is a Hamiltonian cycle of the graph was settled back in 2019 by Alspach, Khodadadpour and Kreher, the author of this talk (and Ted Dobson) became involved in the investigations when the case that the 2-factor consists of two cycles was considered. The main purpose of this talk is to present some more recent results [1], where the situation that the quotient with respect to the 2-factor is a cycle, was investigated thoroughly. The resulting graphs in one of the two essentially different cases represent a very natural generalisation of the Generalised Petersen graphs (as well as of the perhaps somewhat lesser-known Honeycomb toroidal graphs).

Some of the presented results are joint work with Brian Alspach.

References

[1] B. Alspach, P. Šparl, Cubic factor-invariant graphs of cycle quotient type – The alternating case, *European J. Combin.* 120 (2024), Paper No. 103964, 22 pp.