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List of talks

Zombies on the Grid

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In Zombies and Survivors, a set of zombies attempts to eat a lone survivor, Mindy, loose on a given connected graph. The zombies randomly choose their initial location, and during the course of the game, move directly toward Mindy. At each round, they move to the neighbouring vertex that minimizes the distance to Mindy; if there is more than one such vertex, then they choose one uniformly at random. Mindy attempts to escape from the zombies by moving to a neighbouring vertex or staying on her current vertex. The zombies win if eventually one of them eats Mindy by landing on her vertex; otherwise, Mindy wins. The zombie number $z(G)$ of a graph G is the minimum number of zombies needed to play such that the probability that they win is at least $1/2$.

In this paper, we investigate the game played on toroidal grids $C_n \square C_n$. In particular, we show that asymptotically almost surely $z(C_n \square C_n) = \Omega(n/\log^c n)$ for some constant c and that $z(C_n \square C_n) = O(n^{3/2})$.

Algebraic relations for permutons

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Permutons have emerged as a highly successful method for studying structure and certain properties of large permutations. In particular they are closely related to pattern densities. However, they are lacking when considering algebraic properties of the permutations such as cycle structure, order, and permutation compositions. We investigate what can still be said about products of permutations in the permuton limit.

On random regular graphs and the Kim-Vu Sandwich Conjecture

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The random regular graph $G_d(n)$ is selected uniformly at random from all d-regular graphs on n vertices. This model is a lot harder to study than the Erdős-Renyi binomial random graph model $G(n,p)$ as the probabilities of edges being present are not independent. However, in the regime $d \gg \log n$, various graph properties including Hamiltonicity and chromatic number were shown (with hard work) to be the same in $G_d(n)$ as in $G(n,p)$ with $np = d$. This inspired Kim and Vu [2] to conjecture that when $d \gg \log n$ it is possible to 'sandwich' the random regular graph $G_d(n)$ between two Erdős-Renyi random graphs with similar edge density. A proof of the conjecture would immediately imply many results about monotone graph properties of $G_d(n)$ in this dense regime, and would unify all the previous separate hard-won results.

Various authors have proved weaker versions of this conjecture with incrementally improved bounds on d. The previous state of the art was due to Gao, Isaev and McKay [1] who proved the conjecture for $d \gg \log^4 n/(\log\log n)^3$. I will talk about our new improvement of this result.

References

[1] P. Gao, M. Isaev, and B. McKay. Kim-Vu's sandwich conjecture is true for $d \geq \log^4 n$. *arXiv preprint arXiv:2011.09449*, 2020.

[2] J. H. Kim, V. H. Vu, Sandwiching random graphs: universality between random graph models, *Advances in Mathematics* 188(2):444–469, 2004.

Packing edge-disjoint copies of a fixed graph in the random geometric graph

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Fix a graph H. Can we cover almost all the edges of a d-dimensional random geometric graph with edge-disjoint copies of H? Perhaps surprisingly, we will see that for some choices of H, d the answer is “no,” even if the random geometric graph is dense. We also show that the answer is “yes” for many choices of H, d.

Connectivity thresholds for superpositions of Bernoulli random graphs

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Let $G_1, \ldots, G_m$ be independent identically distributed Bernoulli random subgraphs of the complete graph $\mathcal{K}_n$. For $k = 1, 2, \ldots,$ we show the k-connectivity threshold as $n, m \to +\infty$ for the union graph $\cup_{i=1}^m G_i$ defined on the vertex set of $\mathcal{K}_n$. For $k = 2, 3, \ldots$ we observe two different threshold behaviors: one for the unions of cliques and the other one for the (remaining) case where each G_i has a vertex of degree 1 with positive probability. Results for the case $k = 1$ have been reported in [1, 2].

References

[1] D.Ardickas, M.Bloznelis, Connectivity threshold for superpositions of Bernoulli random graphs, *Discrete Math.* 2025, 348, 114684.

[2] M.Bloznelis, D. Marma. R. Vaicekauskas, Connectivity threshold for superpositions of Bernoulli random graphs.II, *Acta Math. Hung.* 2025, 175, 352 - 375.

Loose paths in random ordered hypergraphs

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Consider the random r-uniform hypergraph $H = H^{(r)}(n,p)$. An *ordered loose path* is a sequence of edges $E_1, E_2, \ldots, E_\ell$ of H such that $\max\{j \in E_i\} = \min\{j \in E_{i+1}\}$ for $1 \leq i < \ell$. In this talk we establish fairly tight bounds on the length of the longest ordered loose path in H that hold with high probability.

This is a joint work with Alan Frieze and Wesley Pegden.

Sharp Thresholds Imply Circuit Lower Bounds

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We show that sharp thresholds for Boolean functions imply circuit lower bounds. More formally we show that any Boolean function exhibiting a sharp enough threshold at an arbitrary threshold cannot be computed by Boolean circuits of bounded depth and polynomial size. This verifies a conjecture put forward earlier in the survey by Kalai and Safra. Our result is of particular interest in the sparse random graph setting where the main tool for bounding circuit depth, namely Linial-Mansour-Nisan (LMN) theorem does not apply. We redeem the power of LMN theorem by creating simple dense-to-sparse circuit gadgets. Our result will be illustrated using two models: independent sets in sparse random graphs and random 2-SAT model.

Joint work with Elchanan Mossel (MIT) and Ilias Zadik (Yale University).

References

[1] D.Gamarnik, E.Mossel, I. Zadik, Sharp Thresholds Imply Circuit Lower Bounds: from random 2-SAT to Planted Clique, *Israel Journal of Mathematics, to appear.* 2007 pp.145-161.

Preferential attachment trees with vertex death

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Preferential attachment models are a popular class of random graphs that have received a wealth of attention in the last decades and are often used to model evolving networks. In such models, new vertices are added to the graph sequentially and new vertices are more likely to make connections with existing vertices that have a large degree.

In recent work, we study a general preferential attachment model where vertices can both be added but can also be 'killed'. Such killed vertices can no longer make new connections, whereas 'alive' vertices continue to make new connections. This models evolving networks that can both increase as well as decrease in size.

We focus on *persistence of the maximum degree*: are the oldest alive vertices also the ones with largest degree? We uncover a novel regime in which killing of vertices makes such persistence entirely impossible.

Hitting times and the power of choice for random geometric graphs

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We consider a random geometric graph process where random points $(X_i)_{i\geq 1}$ are embedded consecutively in the d-dimensional unit torus $\mathbb{T}^d$, and every two points at distance at most r form an edge. As $r \to 0$, we confirm that well-known hitting time results for k-connectivity (with $k \geq 1$ fixed) and Hamiltonicity in the Erdős-Rényi graph process also hold for the considered geometric analogue. Moreover, we exhibit a sort of probabilistic monotonicity for each of these properties.

We also study a geometric analogue of the power of choice where, at each step, an agent is given two random points sampled independently and uniformly from $\mathbb{T}^d$ and has to add exactly one of them to the already constructed point set. When the agent is allowed to make their choice with the knowledge of the entire sequence of random points (offline 2-choice), we show that they can construct a connected graph at the first time t when none of the first t pairs of proposed points contains two isolated vertices in the graph induced by $(X_i)_{i=1}^{2t}$, and maintain connectivity thereafter. We also derive analogous results for k-connectivity and Hamiltonicity. This shows that each of the said properties can be attained two times faster (time-wise) and with four times fewer points in the offline 2-choice process compared to the 1-choice process.

In the online version where the agent only knows the process until the current time step, we show that k-connectivity and Hamiltonicity cannot be significantly accelerated (time-wise) but may be realised on two times fewer points compared to the 1-choice analogue.

Recovery of spatial vertex features in noisy SPA model graphs

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The graph matching problem is that of identifying vertices in two graphs that are independent perturbations of a single random graph. Inspired by an approach recently introduced in a paper by Liu and Austern [1], we consider graphs generated via a geometric random graph model. In particular, features associated with each vertex can be interpreted as giving the spatial position of the vertex, and the formation of the graph is informed by the positions of the vertices. Noisy versions of the vertex features are assumed to be given; an important step in graph matching is then that of estimating the true positions of the vertices. We apply this approach to the spatial preferential attachment (SPA) model [2], which generates sparse spatial graphs with a power law degree distribution. We propose a two-step approach to estimating the spatial positions of the vertices, and derive bounds on the noise parameters under which our method is an efficient estimator for the positions.

References

[1] S. Liu, M. Austern, Perfect Recovery for Random Geometric Graph Matching with Shallow Graph Neural Networks, *arXiv:2402.07340* (2025).

[2] W. Aiello, A. Bonato, C. Cooper, J.Janssen, P. Prałat , A spatial web graph model with local influence regions, *Internet Mathematics* 5 (2009), pp. 175–196.

On the threshold for random triangulations inside large convex polygons

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Start with a convex polygon and then add a random graph of edges inside. We will discuss some results concerning the critical point at which a triangulation of the polygon emerges. Joint work with Georgii Zakharov (Oxford) and Maksim Zhukovskii (Sheffield).

Monochromatic matchings in almost-complete and random hypergraphs

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The Ramsey number for matchings in graphs was determined by Cockayne and Lorimer [1] and later extended to hypergraphs by Alon, Frankl, and Lovász [2], who showed that every q-colouring of the complete r-uniform hypergraph on n vertices contains a monochromatic matching of size $\lfloor((n+q-1)/(r+q-1))\rfloor$.

In this talk I will present two extensions of this classical result. The first is a **defect version**, which asserts that every q-colouring of an *almost-complete* uniform hypergraph contains a monochromatic matching of comparable size to that in the complete case. The second is a **transference principle**, which demonstrates that a monochromatic matching of comparable size exists, with high probability, in any q-colouring of a *sparse, random* uniform hypergraph with sufficiently large constant average degree.

The proofs combine methods from extremal set theory with a variant of the weak hypergraph regularity lemma.

References

[1] E. J. Cockayne, P. J. Lorimer, The Ramsey number for stripes, *J. Austral. Math. Soc.* **19** (1975), 252–256.

[2] N. Alon, P. Frankl, L. Lovász, The chromatic number of Kneser hypergraphs, *Trans. Amer. Math. Soc.* **298** (1986), 359–370.

Homogeneous substructures in random ordered uniform matchings

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An ordered r-uniform matching of size n is a collection of n pairwise disjoint r-subsets (edges) of a linearly ordered set of rn vertices. For $n = 2$, such a matching is called an *r-pattern*, as it represents one of $\frac{1}{2}\binom{2r}{r}$ ways two disjoint edges may intertwine. Given a set $\mathcal{P}$ of r-patterns, a *$\mathcal{P}$-clique* is a matching with all pairs of edges belonging to $\mathcal{P}$.

I will present recent results determining asymptotically the largest size of a $\mathcal{P}$-clique in a *random* ordered r-uniform matching, for several classes of sets of patterns $\mathcal{P}$. This is joint work with A. Dudek, J. Grytczuk, and J. Przybyło.