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Labelings of Graphs

Book of abstracts

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List of talks

Group distance magic cubic graphs

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A Γ-*distance magic labeling* of a graph $G = (V, E)$ with $|V| = n$ is a bijection ℓ from V to an Abelian group Γ of order n, for which there exists $\mu \in \Gamma$, such that the weight $w(x) = \sum_{y \in N(x)} \ell(y)$ of every vertex $x \in V$ is equal to μ. In this case, the element μ is called the *magic constant of* G. A graph G is called a *group distance magic* if there exists a Γ-distance magic labeling of G for every Abelian group Γ of order n.

In this talk, we focus on cubic Γ-distance magic graphs as well as some properties of such graphs.

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Self-reverse distance magic labeling

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According to a nonstandard definition introduced in 2021, a distance magic labeling ℓ of a regular graph of order n is a bijection from its vertex set to the set of integers of the arithmetic progression from $1-n$ to $n-1$ with common difference 2, such that the sum of the labels of the neighbors of each vertex is zero. Such a labeling is called *self-reverse* if, for any pair of vertices u and v, u is adjacent to v if and only if the vertices with labels $-\ell(u)$ and $-\ell(v)$ are adjacent.

In this talk, we present the motivation for studying self-reverse distance magic labelings. We focus on self-reverse distance magic labelings in the case of tetravalent graphs providing several examples and a complete classification of all orders for which a tetravalent graph admitting such a labeling exists. The classification is obtained via a novel construction that produces a (tetravalent) distance magic graph from two given (tetravalent) distance magic graphs. We also discuss the existence of graphs admitting a self-reverse distance magic labeling among some well-known families of tetravalent graphs.

Pushing for Irregularity

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Let G be a simple graph with maximum degree $\Delta(G)$ and no component of order 2. Bensmail, Marcille, and Orenga [1] introduce the notion of a *pushing scheme* $\rho : V(G) \to \mathbb{N}_0$ with induced edge labeling

$$\ell : E(G) \to \mathbb{N}, \quad uv \mapsto 1 + \rho(u) + \rho(v).$$

ρ should be chosen such that the induced vertex labeling

$$\sigma : V(G) \to \mathbb{N}_0, \quad v \mapsto \sum_{u \in N_G(v)} \ell(uv)$$

is a proper vertex coloring. Bensmail et al. conjecture that such a
$\rho : V(G) \to \{0, 1, \ldots, \Delta(G)\}$ exists for every graph G. We prove their conjecture for a few graph classes [2].

References

[1] J. Bensmail, C. Marcille, and M. Orenga, Pushing Vertices to Make Graphs Irregular, hal-04810955.

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On rainbow caterpillars

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Given a finite Abelian group $(A, +)$, consider a tree T with $|A|$ vertices. The labeling $f\colon V(T) \to A$ of the vertices of some graph G induces an edge labeling in G, thus the edge uv receives the label $f(u) + f(v)$. The tree T is A-rainbow colored if f is a bijection and edges have different colors. In this paper, we give necessary and sufficient conditions for a caterpillar with three spine vertices to be A-rainbow, when A is an elementary p-group.

References

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