10th Cracow Conference on Graph Theory

Graph Product

Book of abstracts

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List of talks

Some recent results on modular product

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A product of two graphs G and H has a vertex set $V(G) \times V(H)$. For edges, we consider three objects in every factor for the definition of an edge in a product: either a vertex or an edge or a non-edge. Combining the mention objects from one factor with the same objects in the other factor gives eight different possibilities (notice that vertex by vertex must be ignored to avoid loops) to have an edge or a non-edge in a product. So, all together $2^8 = 256$ different graph products exists and four of them—Cartesian $G \Box H$, strong $G \boxtimes H$, direct $G \times H$ and lexicographic $G \circ H$—gain a special status and are called standard products.

Among all graph products only 10 are associative and commutative. We join them into pairs of a product $G * H$ together with its complementary product $G \overline{*} H \overset{\text{def}}{=} \overline{\overline{G} * \overline{H}}$ where $\overline{G}$ is the complement of a graph G. Six of them represent either a standard product (Cartesian, strong or direct) or its complementary product, next two are empty and complete product and finally modular product $G \diamond H$, where $E(G \diamond H) = E(G \Box H) \cup E(G \times H) \cup E(\overline{G} \times \overline{H})$, and its complementary product.

We present several recent results on distance [2], domination number [1] and independence sets of modular products with respect to some properties of their factors.

References

[1] S. Bermudo, I. Peterin, J. Sedlar, R. Škrekovski, Domination number of modular product graphs, Comput. Applied Math. 44(1) (2025) #65.

[2] C.X. Kang, A. Kelenc, I. Peterin, E.Yi, On Distance and Strong Metric Dimension of the Modular Product, Bulletin Malaysian Math. Sciences Soc. 48(1) (2025) #20.

Integrity of grids

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The integrity of a graph $G = (V, E)$ is defined as the smallest sum $|S| + m(G - S)$, where S is a subset of the set V, and $m(H)$ denotes the order of the largest component of the graph H.

Benko, Ernst, and Lanphier provided and proved an asymptotic bounds for planar graphs in terms of the order of the graph. We prove asymptotic results concerning two-dimensional grid-graphs.

References

[1] D. Benko, C. Ernst, D. Lanphier, Asymptotic bounds on the integrity of graphs and separator theorems for graphs, *SIAM Journal on Discrete Mathematics* 23 2009 pp.265–277.

[2] A. Żak, A note of integrity, *Discrete Applied Mathematics* 341 2023, pp.55–59.

Construction of k-matchings in graph products

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The study of classical graph invariants—such as chromatic, domination, and independence numbers—in graph products has received significant attention. Here, we focus on variations of matchings in the four standard graph products: Cartesian, strong, direct and lexicographic. Specifically, we define a subset $M \subseteq E$ of a graph $G = (V, E)$ as a *k-matching* if the edges in M induce a k-regular subgraph of G.

Summarizing results in [1], we present explicit constructions of k-matchings in graph products $G \star H$, utilizing k_G-matchings M_G and k_H-matchings M_H from the factor graphs G and H. Although these constructions do not always yield maximum k-matchings for the product, they achieve the largest possible size among all k-matchings that are weak-homomorphism preserving – meaning that matched edges in the product never project onto unmatched edges in the factors.

References

[1] A. Lindeberg, M. Hellmuth. Construction of k-matchings in graph products. *Art Discrete Appl. Math.* 6(2) (2022). DOI:10.26493/2590-9770.1462.b03.

Planar and polyhedral graphs as Kronecker and Sierpiński products

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We will discuss graph products that are planar/polyhedral. The first part of the talk focuses on the Kronecker (direct, tensor) product [2], Figure 1. We also consider simultaneous products [3].

The second part of the talk [4] is on the Sierpiński product, recently introduced in [1].

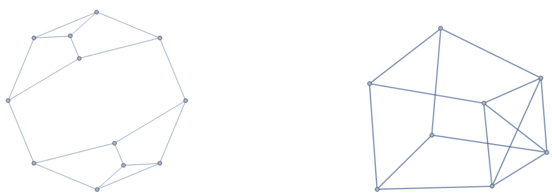

Figure 1: Illustrations of a planar (left) and a non-planar (right) factor for planar, 3-connected Kronecker products.

References

[1] J.Kovic, T.Pisanski, S.S.Zemljic, A.Zitnik. 'The Sierpiński product of graphs'. *Ars Math. Contemp.* 2023, 23(1).

[2] R.Maffucci, 'Classification and Construction of Planar, 3-Connected Kronecker Products', *arXiv:2402.01407*.

[3] R.De March, R.Maffucci, 'Cancellation and regularity for planar, 3-connected Kronecker products' *arXiv:2411.13473*.

[4] R.Maffucci, 'Regularity and separation for Sierpiński products of graphs', *arXiv:2506.16864*.