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General

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3-colourability, diamonds and butterflies

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The 3-colourability problem is an NP-complete problem which remains NP-complete for graphs with maximum degree four, for claw-free graphs, and even for (claw,diamond)-free graphs. In this talk we will consider induced subgraphs, among them are the *claw* ($K_{1,3}$), the *diamond* (the graph $K_4 - e$), the *butterfly* (two triangles sharing a vertex), and the generalized *net* $N_{i,j,k}$ (a triangle with three attached paths with i, j, k edges).

Our main result is a complete characterization of all 3-colourable $(claw, diamond, H)$-free graphs for $H \in \{N_{1,1,1}, N_{1,1,2}, N_{1,2,2}, N_{2,2,2}\}$. We will present a description of all non 3-colourable $(claw, diamond, H)$-free graphs for $H \in \{N_{1,1,1}, N_{1,1,2}, N_{1,2,2}, N_{2,2,2}\}$ in terms of butterflies. Moreover, we will show extensions of this characterization to larger graph classes.

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On Mycielskians of digraphs

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Let D be a digraph on vertices $v_1, \ldots, v_n$. The *Mycielskian* of D, denoted $M(D)$, is obtained from D by adding an independent set of vertices $V' = \{v'_1, \ldots, v'_n\}$ and one extra vertex x. For every arc (v_i, v_j) of D, the arcs (v_i, v'_j) and (v'_i, v_j) are added, and finally arcs from x to all vertices of V' are included. In a natural way, a sequence of digraphs $\{M_p(D)\}_{p \geq 0}$ is defined by $M_0(D) = D$, $M_1(D) = M(D)$, and $M_p(D) = M(M_{p-1}(D))$ for $p \geq 2$, where $M_p(D)$ is called the *p-th Mycielskian* of D.

For a digraph D, a set S of arcs is a *feedback arc set* if $D - S$ is acyclic, and the minimum size of such a set is denoted by $\tau_1(D)$. In this talk we focus on the parameter $\tau_1(M_p(D))$, as well as on the maximum number of arc-disjoint directed cycles in $M_p(D)$, denoted by $\nu_1(M_p(D))$, which is closely related to $\tau_1(M_p(D))$.

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Further progress on Wojda's conjecture

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Two digraphs of order n are said to pack if they can be found as edge-disjoint subgraphs of the complete digraph of order n. It is well established that if the sum of the sizes of the two digraphs is at most $2n-2$, then they pack, with this bound being sharp. However, it is sufficient for the size of the smaller digraph to be only slightly below n for the sum of their sizes to significantly exceed this threshold while still guaranteeing the existence of a packing.

In 1985, Wojda conjectured that for any $2 \leq m \leq n/2$, if one digraph has size at most $n-m$ and the other has size less than $2n - \lfloor n/m \rfloor$, then the two digraphs pack. It was previously known that this conjecture holds for $m = \Omega(\sqrt{n})$. We confirm it for $m \geq 26$.

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On coarse tree decompositions and coarse balanced separators

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It is known that there is a linear dependence between the treewidth of a graph and its *balanced separator number*: the smallest integer k such that for every weighing of the vertices, the graph admits a balanced separator of size at most k. We investigate whether this connection can be lifted to the setting of coarse graph theory, where both the bags of the considered tree decompositions and the considered separators should be coverable by a bounded number of bounded-radius balls.

As the first result, we prove that if an n-vertex graph G admits balanced separators coverable by k balls of radius r, then G also admits tree decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$ such that:

- in $\mathcal{T}_1$, every bag can be covered by $\mathcal{O}(k \log n)$ balls of radius r; and
- in $\mathcal{T}_2$, every bag can be covered by $\mathcal{O}(k^2 \log k)$ balls of radius $r(\log k + \log \log n + \mathcal{O}(1))$.

As the second result, we show that if we additionally assume that G has doubling dimension at most m, then the functional equivalence between the existence of small balanced separators and of tree decompositions of small width can be fully lifted to the coarse setting.

Constructing algebraic expressions for lattice-structured digraphs

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We investigate relationship between algebraic expressions and *labeled two-terminal directed acyclic graphs* (labeled *st-dags*), in which each edge carries a unique label. An algebraic expression is referred to as an *st-dag expression* if it is algebraically equivalent to the sum of edge-label products over all spanning paths of the st-dag. The st-dags considered here are based on lattice structure and have $m \times n$ vertices (m rows and n columns). Examples are shown in Figure 1: (a) *grid graph* $G_{m,n}$, (b) *triangular grid* $T_{m,n}$, (c) *king graph* $K_{m,n}$. We treat m as a constant representing the graph's *depth*, while n determines its *size*. Our objective is to simplify the expressions for these graphs. To this end, we apply *backtracking* and *decomposition* methods (*BM* and *DM*, respectively) which generate expressions for these graphs, and we estimate the lengths of the generated expressions as functions of n. BM produces expressions of length $O\left(n^m\right)$ for both $G_{m,n}$ and $T_{m,n}$, and of exponential length in n for $K_{m,n}$. In contrast, DM yields more compact expressions: length $O\left(n \log^{m-1} n\right)$ for both $G_{m,n}$ and $T_{m,n}$ and length $O\left(n^{\log_2(4m-2)}\right)$ for $K_{m,n}$.

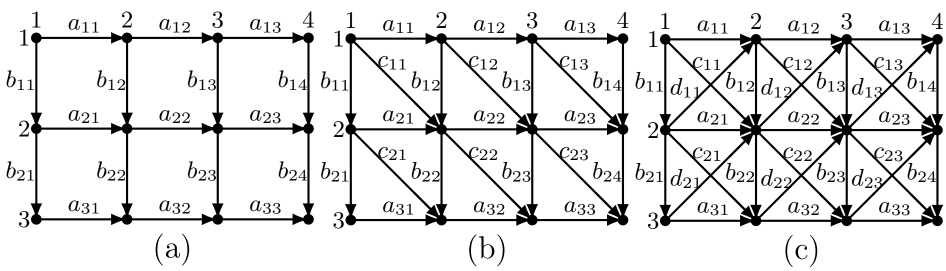

Figure 1: Lattice-structured digraphs.

A generalization of an ear decomposition and k-trees in highly connected star-free graphs

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In this talk, we introduce a generalized version of an ear decomposition, called a j-spider decomposition, for j-connected star-free graphs with $j \geq 2$. Its application enables us to improve a previousely known sufficient condition for the existence of a k-tree in highly connected star-free graphs, where a k-tree is a spanning tree in which every vertex is of degree at most k. More precisely, we show that every j-connected $K_{1,j(k-2)+2}$-free graph has a k-tree for $k \geq j$, thereby improving a classical result of Jackson and Wormald [1] for $k \geq j$. Our approach differs from previous studies based on toughness-type arguments and instead relies on both a j-spider decomposition and a factor theorem related to Hall's marriage theorem.

This talk is based on the paper [2].

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H-kernels in 3-quasi-transitive digraphs

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Let H be a digraph possibly with loops and D a digraph without loops whose arcs are colored with the vertices of H (D is said to be an H-colored digraph). A directed path P in D is said to be an H-path if and only if the consecutive colors encountered on P form a directed walk in H. An H-kernel of an H-colored digraph D is a subset of vertices of D, say N, such that for every pair of different vertices in N there is no H-path between them, and for every vertex u in $V(D) \setminus N$ there exists an H-path in D from u to N. D is said to be 3-quasi-transitive if for every pair of vertices u and v of D, the existence of a directed path of length 3 from u to v implies that $\{(u,v),(v,u)\} \cap A(D) \neq \emptyset$. In this talk we show a result regarding the existence of H-kernels in 3-quasi-transitive digraphs; mainly the existence of H-kernels is guaranteed by means of sufficient conditions on the directed cycles of length 3 and 4.

Representing Distance-Hereditary Graphs with Trees

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Cographs are precisely the undirected graphs that can be represented by a vertex-labelled tree, that is, a pair (T, t), where T is a rooted tree and t is a labelling of the vertices of T into the set $\{0, 1\}$. Specifically, we say that the pair (T, t) explains G if T has leaf set $V(G)$ and, for any two distinct vertices x and y of G, x and y are joined by an edge in G if and only if the least common ancestor of x and y in T has label 1 via t.

Recently [1], the class of arboreal networks was introduced as a generalization of rooted trees. Arboreal networks are directed, acyclic graphs whose underlying, undirected graph is a tree. Intuitively, they are rooted trees which can have more than one root. This led to the question of characterizing those undirected graphs G that can be explained by a vertex-labelled arboreal network (N, t), in the same way cographs are explained by vertex-labelled trees. Interestingly, this is a well known and well studied class of graphs [2].

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