10th Cracow Conference on Graph Theory

Domination Graph Theory

Book of abstracts

Cracow, September 22-26, 2025

https://10ccgt.agh.edu.pl/

List of talks

Domination type parameters in 3-regular and 4-regular graphs

Michael A. Henning[(1)]

[(1)] University of Johannesburg, Johannesburg, South Africa

A set S of vertices in a graph G is a dominating set if every vertex in $V(G) \setminus S$ is adjacent to a vertex in S. The domination number, $\gamma(G)$, of G is the minimum cardinality among all dominating set in G. We discuss best possible upper bounds on domination-type parameters in cubic graphs. Among other results, we show that if G is a cubic graph of order n, then $\gamma_{t2}(G) \le \frac{2}{5}n$ and $\gamma_r(G) \le \frac{2}{5}n$, where $\gamma_{t2}(G)$ and $\gamma_r(G)$ denote the semitotal and restrained domination numbers, respectively. The $\frac{1}{3}$-conjecture for domination in 4-regular graphs states that if G is a 4-regular graph of order n, then $\gamma(G) \le \frac{1}{3}n$. We prove this conjecture when G has no induced 4-cycle. A thorough treatise on dominating sets can be found in [1, 2, 3].

References

[1] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning (eds), *Topics in Domination in Graphs*. Series: Developments in Mathematics, Vol. 64, Springer, Cham, 2020. viii + 545 pp.

[2] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning (eds), *Structures of Domination in Graphs*. Series: Developments in Mathematics, Vol. 66, Springer, Cham, 2021. viii + 536 pp.

[3] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning, *Domination in Graphs: Core Concepts* Series: Springer Monographs in Mathematics, Springer, Cham, 2023. xx + 644 pp.

On proper secondary and multiple dominating sets

Paweł Bednarz[(1)], Adrian Michalski[(1)]

[(1)] Rzeszow University of Technology, Rzeszów, Poland

Let $k \geq 1$ be an integer. A subset $D \subset V(G)$ is (1,k)-dominating if for every vertex $v \in V(G) \setminus D$ there exist vertices $u, w \in D$ such that $uv \in E(G)$ and $d_G(v, w) \leq k$. If $k = 1$, then we obtain the definition of (1,1)-dominating sets, which are also known as 2-dominating sets. If $k = 2$, then we have the concept of (1,2)-dominating sets.

In [1] Michalski et al. introduced the concept of proper (1,2)-dominating sets to distinguish (1,2)-dominating sets from (1,1)-dominating sets. Formally, a *proper* (1,2)-*dominating set* is a (1,2)-dominating set that is not (1,1)-dominating. Basing on this idea, we considered proper (1,3)-dominating sets. Moreover, in [2, 3] proper l-dominating sets i.e. l-dominating sets which are not $(l + 1)$-dominating were defined and studied.

In this talk we present some results concerning proper secondary and multiple dominating sets, in particular we focus on the problem of their existence. Moreover, we show relations between parameters of these types of domination.

References

[1] A. Michalski, I. Włoch, M. Dettlaff, M. Lemańska, *On proper (1,2)-dominating sets*, Mathematical Methods in the Applied Sciences, 45(11), (2022), 7050–7057.

[2] P. Bednarz, M. Pirga, *On proper 2-dominating sets in graphs*, Symmetry, 16, (2024), 296.

[3] D. Chen, S. Cai, *Proper 3-Dominating Sets in Graphs*, Mathematics, 13, (2025), 1960.

Isolation of graphs

Peter Borg[(1)]

[(1)] University of Malta, Msida, Malta

Given a set $\mathcal{F}$ of graphs, we call a copy of a graph in $\mathcal{F}$ an $\mathcal{F}$-*graph*. The $\mathcal{F}$*-isolation number* of a graph G, denoted by $\iota(G, \mathcal{F})$, is the size of a smallest subset D of the vertex set of G such that the closed neighbourhood $N[D]$ of D intersects the vertex sets of the $\mathcal{F}$-graphs contained by G (equivalently, $G - N[D]$ contains no $\mathcal{F}$-graph). When $\mathcal{F}$ consists of a 1-clique, $\iota(G, \mathcal{F})$ is the *domination number* of G. When $\mathcal{F}$ consists of a 2-clique, $\iota(G, \mathcal{F})$ is the *vertex-edge domination number* of G. The study of the general $\mathcal{F}$-isolation problem was introduced by Caro and Hansberg [4] in 2017. This study is expanding very rapidly. A brief account of its development and of the speaker's recent work in this field [1, 2, 3] will be provided.

References

[1] P.Borg, Isolation of regular graphs, stars and k-chromatic graphs, arXiv:2303.13709 [math.CO].

[2] P. Borg, Isolation of regular graphs and k-chromatic graphs, *Mediterr. J. Math.* 21 (2024) paper 148.

[3] P. Borg, Proof of a conjecture on isolation of graphs dominated by a vertex, *Discrete Appl. Math.* 371 (2025) pp.247-253.

[4] Y.Caro, A.Hansberg, Partial domination - the isolation number of a graph, *Filomat* 31 (2017) pp.3925-3944.

2-Rainbow Independent Domination in Complementary Prisms

Dragana Božović[1], Gordana Radić[1,2], Aleksandra Tepeh[1,2]

[1] University of Maribor, Faculty of Electrical Engineering and Computer Science, Koroška cesta 46, 2000 Maribor, Slovenia
[2] Institute of Mathematics, Physics and Mechanics, Jadranska ulica 19, 1000 Ljubljana, Slovenia

A function f that assigns values from the set $\{0, 1, 2\}$ to each vertex of a graph G is called a 2-rainbow independent dominating function, if the vertices assigned the value 1 form an independent set, the vertices assigned the value 2 form another independent set, and every vertex to which 0 is assigned has at least one neighbor in each of the mentioned independent sets. The weight of this function is the total number of vertices assigned nonzero values. The 2-rainbow independent domination number of G, $\gamma_{\mathrm{ri2}}(G)$, is the minimum weight of such a function.

We study the 2-rainbow independent domination number of the complementary prism $G\overline{G}$ of a graph G, which is constructed by taking G and its complement $\overline{G}$, and then adding edges between corresponding vertices. We provide tight bounds for $\gamma_{\mathrm{ri2}}(G\overline{G})$, and characterize graphs for which the lower bound, i.e. $\max\{\gamma_{\mathrm{ri2}}(G), \gamma_{\mathrm{ri2}}(\overline{G})\} + 1$, is attained.

References

[1] S. Brezovnik, T. Kraner Šumenjak, Complexity of k -rainbow independent domination and some results on the lexicographic product of graphs, *Appl. Math. Comp.* **349** (2019) 214–220.

[2] B. Gabrovšek, A. Peperko, J. Žerovnik, On the 2-rainbow independent domination numbers of some graphs, *Cent. Eur. J. Oper. Res.* **31(1)** (2023) 1–15.

[3] R. Hammack, W. Imrich, and S. Klavžar, *Handbook of Product Graphs: Second Edition*, CRC Press, Boca Raton, FL, 2011.

[4] T.W. Haynes, M.A. Henning, P.J. Slater and L.C. Van Der Merwe, The complementary product of two graphs, Bull. Instit. Combin. Appl. **51** (2007), 21–30.

[5] T. Kraner Šumenjak, D. F. Rall, A. Tepeh, On k-rainbow independent domination in graphs, *Appl. Math. Comp.* **333** (2018) 353–361.

[6] X. Shi, P. Wu, Z. Shao, V. Samodivkin, S.M. Sheikholeslami, M. Soroudi, and S. Wang, Changing and unchanging 2-rainbow independent domination, *IEEE Access* **7** (2019) 72604–72612.

[7] E. Zhu, An improved Nordhaus–Gaddum-type theorem for 2-rainbow independent domination number, *Mathematics* **9** (2021) 402.

An algorithmic proof for the domination number of grid graphs

Yao-Ting Shen[(1)], Chiuyuan Chen[(1)]

[(1)] National Yang Ming Chiao Tung University, Hsinchu, Taiwan

The *domination number* $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set of G. Let the notion $[i]$ denote the set $\{1, 2, \ldots, i\}$. Let $G_{m,n}$ denote the complete (m, n) grid; i.e., the vertex set of $G_{m,n}$ is $[m] \times [n]$, and two vertices (i, j) and (i', j') are adjacent if $|i - i'| + |j - j'| = 1$. Reference [1] proves that for every $16 \leq m \leq n$, $\gamma(G_{m,n}) = \left\lfloor \frac{(m+2)(n+2)}{5} \right\rfloor - 4$ and thus concludes the calculation of $\gamma(G_{m,n})$ of all (m, n) grid graphs. In this study (a preliminary version was in [2]), we consider the charging pad deployment problem (CPDP) for wireless rechargeable sensor networks with grid topology $\mathcal{G}_{m,n}$. CPDP aims to find a deployment of charging pads for the unmanned aerial vehicle (UAV) so that every sensor node is covered by at least one pad and the number of pads is as small as possible. Note that $\mathcal{G}_{m,n} \cong G_{m+1,n+1}$. We show that when the grid length $\mathcal{L}$ satisfies $\frac{1}{\sqrt{2}} d_\theta < \mathcal{L} \leq \frac{2}{\sqrt{5}} d_\theta$, CPDP is related to finding $\gamma(\mathcal{G}_{m,n})$, where d_θ is the maximum flying distance of the UAV when its energy is θ (a pre-defined energy threshold). We propose a charging pad deployment scheme for $\mathcal{G}_{m,n}$ and prove that for every $15 \leq m \leq n$, our scheme uses the least number of pads; this provides an algorithmic proof for the domination number of grid graphs.

References

[1] D. Gonçalves, A. Pinlou, M. Rao, and S. Thomassé, The domination number of grids, *SIAM J. Discrete Math.*, vol. 25, no. 3, pp. 1443–1453, 2011.

[2] Y.-T. Shen, *Three charging pad deployment schemes for wireless rechargeable sensor networks with grid topology*, (Master thesis), National Yang Ming Chiao Tung University, Hsinchu, Taiwan, 2023.

2-Domination edge subdivision in trees

M. Dettlaff[(1)], M.A. Henning[(2)], M. Lemańska[(3)], A. Roux[(4)], J. Topp[(5)]

[(1)] University of Gdańsk, Gdańsk, Poland
[(2)] University of Johannesburg, Johannesburg, South Africa
[(3)] Gdańsk University of Technology, Gdańsk, Poland
[(4)] Stellenbosch University, Stellenbosch, South Africa
[(5)] University of Applied Sciences, Elbląg, Poland

A set S of vertices in a graph G is a 2-dominating set of G if every vertex not in S has at least two neighbors in S, where two vertices are neighbors if they are adjacent. The 2-domination number of G, denoted by $\gamma_2(G)$, is the minimum cardinality among all 2-dominating sets in G. A γ_2-set of G is a 2-dominating set of G of cardinality $\gamma_2(G)$. The 2-domination subdivision number of G, denoted by $\mathrm{sd}_2(G)$, is the minimum number of edges which must be subdivided in order to increase the 2-domination number. If T is a tree of order $n \geq 3$, then $\mathrm{sd}_2(T) \leq 2$. In [1] we show that $\mathrm{sd}_2(T) = 1$ if and only if the set of vertices that belong to no γ_2-set of G is nonempty. A graph G is γ_2-q-critical if q is the least number such that for every subset of edges S of cardinality q, the graph produced by subdivision of S has a greater 2-domination number. If T is a γ_2-q-critical tree of order $n \geq 3$, then we prove that $q \leq n-2$. Among other results, we characterize γ_2-q-critical trees when q is large, namely $q \in \{n-4, n-3, n-2\}$. We also characterize γ_2-1-critical trees [1] and γ_2-2-critical trees [2].

References

[1] M.Dettlaff, M.A.Henning, M.Lemańska, A.Roux, J.Topp, 2-Domination edge subdivision in trees, *Australas. J. Combin.* to appear.

[2] M.Dettlaff, M.A.Henning, M.Lemańska, A.Roux, 2-Domination critical trees upon edge subdivision, *Australas. J. Combin.* 92(3) (2025), 357—381.

Complexity of Defensive Domination

Steven Chaplick[(1)], Grzegorz Gutowski[(2)], Tomasz Krawczyk[(3)]

[(1)] Maastricht University
[(2)] Jagiellonian University
[(3)] Warsaw University of Technology

In a graph G, a *k-attack* A is any set of at most k vertices and *ℓ-defense* D is a set of at most ℓ vertices. We say that defense D *counters* attack A if each $a \in A$ can be matched to a distinct *defender* $d \in D$ with a equal to d or a adjacent to d in G. In the *defensive domination problem*, we are interested in deciding, for a graph G and positive integers k and ℓ given on input, if there exists an ℓ-defense that counters every possible k-attack on G. Defensive domination is a natural resource allocation problem and can be used to model network robustness and security, disaster response strategies, and redundancy designs.

The defensive domination problem is naturally in the complexity class Σ_2^{P}. The problem was known to be NP-hard in general, and polynomial-time algorithms were found for some restricted graph classes. In this note, we prove that the defensive domination problem is Σ_2^{P}-complete.

We also introduce a natural variant of the defensive domination problem in which the defense is allowed to be a multiset of vertices. This variant is also Σ_2^{P}-complete, but we show that it admits a polynomial-time algorithm in the class of interval graphs. A similar result was known for the original setting in the class of proper interval graphs.

References

[1] Steven Chaplick, Grzegorz Gutowski, and Tomasz Krawczyk. A Note on the Complexity of Defensive Domination, 2025. `https://arxiv.org/abs/2504.14390`.

On rainbow domination regular graphs

Boštjan Kuzman(1)

(1) University of Ljubljana, Slovenia

In [1], a d-regular graph X is called d-rainbow domination regular or d-RDR, if its d-rainbow domination number $\gamma_{rd}(X)$ attains the lower bound $n/2$ for d-regular graphs, where n is the number of vertices. In [2], some combinatorial constructions to construct new d-RDR graphs from existing ones are given and two general criteria for a vertex-transitive d-regular graph to be d-RDR are proven. A list of vertex-transitive 3-RDR graphs of small orders is then produced and their partial classification into families of generalized Petersen graphs, honeycomb-toroidal graphs and a specific family of Cayley graphs is given by investigating the girth and local cycle structure of these graphs. In the talk, some more recent results and open problems on d-RDR graphs will be presented.

References

[1] KUZMAN, Boštjan. On k-rainbow domination in regular graphs. *Discrete applied mathematics*. 2020, vol. 284, pp. 454 – 464.

[2] KUZMAN, Boštjan. On cubic rainbow domination regular graphs. *Discrete applied mathematics*. 2025, vol. 373, pp. 26 – 38.

Paired versus double domination in forbidden graph classes

Michael A. Henning[1], Paras Vinubhai Maniya[2], Dinabandhu Pradhan[2]

[1] University of Johannesburg, Auckland Park, South Africa
[2] Indian Institute of Technology (ISM), Dhanbad, India

A set D of vertices in a graph G is a dominating set of G if every vertex not in D has a neighbor in D, where two vertices are neighbors if they are adjacent. If the dominating set D of G has the additional property that the subgraph induced by D contains a perfect matching (not necessarily as an induced subgraph), then D is a paired dominating set of G. The paired domination number of G, denoted by $\gamma_{pr}(G)$, is the minimum cardinality of a paired dominating set of G. A set $D \subseteq V(G)$ is a double dominating set of G if every vertex in $V(G) \setminus D$ has at least two neighbors in D, and every vertex in D has a neighbor in D. The double domination number of G, denoted by $\gamma_{\times 2}(G)$, is the minimum cardinality of a double dominating set of G. Chellali and Haynes [2] showed that if G is a claw-free graph without isolated vertices, then the paired domination number of G is at most the double domination number of G. In this paper, we show that if G is a H-free graph for some $H \in \{P_5, 2K_2 \cup K_1, \text{fork}\}$ without isolated vertices, then $\gamma_{pr}(G) \leq \gamma_{\times 2}(G)$.

References

[1] M. Blidia, M. Chellali, T. W. Haynes, Characterizations of trees with equal paired and double domination numbers. *Discrete Math.* **306** (2006), 1840–1845.

[2] M. Chellali and T. W. Haynes, On paired and double domination in graphs. *Util. Math.* **67** (2005), 161–71.

Perfect (1,2)-Dominating Sets in Graphs with a Few Large-Degree Vertices

M. Pirga(1)

(1) Rzeszow University of Technology, Poland

Let $k \geq 1$ be an integer. A subset $D \subseteq V(G)$ is a $(1,k)$-dominating set if for every vertex $v \in V(G) \setminus D$ there exist $u, w \in D$ such that $uv \in E(G)$ and $d_G(v,w) \leq k$. The concept of $(1,2)$-dominating sets was introduced in [1] and further studied in [2, 3], where A. Michalski defined a *proper* $(1,2)$*-dominating set* as a $(1,2)$-dominating set that is not $(1,1)$-dominating. Based on this idea, in [4] we introduced a *perfect* $(1,2)$*-dominating set* (shortly $(1,2)$-PDS) as a $(1,2)$-dominating set in which every vertex outside D is adjacent to exactly one vertex of D.

In this talk, we investigate the existence of $(1,2)$-PDS in graphs containing at most two vertices of maximum degree. In particular, we provide a complete characterization for the cases $\Delta(G) = n - 1$ and $\Delta(G) = n - 2$.

References

[1] S. M. Hedetniemi, S. T. Hedetniemi, J. Knisely, D. F. Rall, *Secondary domination in graphs*, AKCE International Journal of Graphs and Combinatorics, 5, (2008), 103–115.

[2] A. Michalski, *Secondary dominating sets in graphs and their modification*, Book of abstracts, The 7th Gdańsk Workshop on Graph Theory, 2019.

[3] A. Michalski, I. Włoch, M. Dettlaff, M. Lemańska, *On proper (1,2)-dominating sets*, Mathematical Methods in the Applied Sciences, 45(11), (2022), 7050–7057.

[4] U. Bednarz, M. Pirga, *(1, 2)-PDS in graphs with the small number of vertices of large degrees*, Opuscula Mathematica 45, no. 1 (2025), 53-62.

Disjoint dominating and 2-dominating sets in graphs: Hardness and Approximation results

Soumyashree Rana(1), Sounaka Mishra(2), B. S. Panda(1)

(1) Department of Mathematics, Indian Institute of Technology Delhi, India
(2) Department of Mathematics, Indian Institute of Technology Madras, India

A set $D \subseteq V$ of a graph $G = (V, E)$ is a dominating set of G if each vertex $v \in V \setminus D$ is adjacent to at least one vertex in D, whereas a set $D_2 \subseteq V$ is a 2-dominating (double dominating) set of G if each vertex $v \in V \setminus D_2$ is adjacent to at least two vertices in D_2. A graph G is a DD_2-graph if there exists a pair (D, D_2) of disjoint dominating set and 2-dominating set of G. In [1], several open problems related to DD_2-graphs were posed. In this paper, we answers some of these problems and present the following results: we provide an approximation algorithm for the problem of determining a minimal spanning DD_2-graph of minimum size (Min-DD_2) with an approximation ratio of 3; a minimal spanning DD_2-graph of maximum size (Max-DD_2) with an approximation ratio of 3; and the smallest number of edges which when added to a non-DD_2-graph results in a minimal spanning DD_2-graph for any graph (Min-to-DD_2) with an $O(\log n)$ approximation ratio. Additionally, we prove that Min-DD_2 and Max-DD_2 are APX-complete for graphs with maximum degree 4. Moreover, for a 3-regular graph, we show that Min-DD_2 and Max-DD_2 are approximable within a factor of 1.8 and 1.5 respectively.

References

[1] M.Miotk, J.Topp, and P.Żyliński. *Disjoint dominating and 2-dominating sets in graphs*, Discrete Optimization, 35:100553, (2020).

Induced cycles vertex number vs. $(1,2)$-domination in cubic graphs

Rija Erveš[(1)], Aleksandra Tepeh[(2,3)]

[(1)] University of Maribor, FCETEA, Slovenia
[(2)] University of Maribor, FEECS, Slovenia
[(3)] Institute of Mathematics, Physics and Mechanics, Slovenia

A $(1,2)$-dominating set in a graph G is a set S such that every vertex outside S has at least one neighbor in S, and each vertex in S has at least two neighbors in S. The $(1,2)$-domination number, $\gamma_{1,2}(G)$, is the minimum size of such a set, while $c_{\rm ind}(G)$ is the cardinality of the largest vertex set in G that induces one or more cycles. In this paper, we initiate the study of a relationship between these two concepts and discuss how establishing such a connection can contribute to solving a conjecture on the lower bound of $c_{\rm ind}(G)$ for cubic graphs. We also establish an upper bound on $c_{\rm ind}(G)$ for cubic graphs and characterize graphs that achieve this bound.

References

[1] R. Erveš, A. Tepeh , Induced cycles vertex number and $(1,2)$-domination in cubic graphs, *Applied Math. Comput.* (to appear).