# 10th Cracow Conference on Graph Theory

## Design Theory

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# List of talks

# Shiftable Heffter Spaces

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The notion of a Heffter array [1] is equivalent to a pair of orthogonal Heffter systems. In [2, 3] we proved the existence of a set of $r$ mutually orthogonal Heffter systems for any $r$. Such a set is equivalent to a resolvable partial linear space of degree $r$ whose parallel classes are Heffter systems: we call such a design a *Heffter space*. In this talk we focus on *shiftable* Heffter spaces [4] presenting a direct construction, making use of pandiagonal magic squares, and a recursive one.

## References

[1] D.S. Archdeacon, Heffter arrays and biembedding graphs on surfaces, *Electron. J. Combin.* 2015 #P1.74.

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# On the Buratti-Horak-Rosa Conjecture for Small Supports

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Label the vertices of the complete graph $K_v$ with the distinct elements of $\mathbb{Z}_v$ and define the *length* $\ell$ of each edge as the cyclical distance between labels of its end-vertices. A Hamiltonian path through $K_v$ is called a *realization* of a given multiset $L$ if its edge labels are $L$. The *Buratti-Horak-Rosa Conjecture* is that there is a realization for a multiset $L$ if and only if for any divisor $d$ of $v$ the number of multiples of $d$ in $L$ is at most $v - d$.

The toroidal lattice of vertices associated with each multiset was shown to be useful for constructing special types of realizations, the concatenations of which yield realizations for larger multisets [1, 2, 3, 4]. We will present our recent constructions yielding “standard linear realizations" for multisets with support of the form $\{1, x, y\}$ whenever the number of 1-edges is at least $\max(x, y) + \gcd(x, y) - 1$. These constructions considerably extend the parameters for which the conjecture is known to hold.

## References

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[2] O. Ağırseven, M. A. Ollis, A Coprime Buratti-Horak-Rosa Conjecture and grid-based linear realizations, *submitted.*

[3] O. Ağırseven, M. A. Ollis, Construction techniques for linear realizations of multisets with small support, *accepted.*

[4] O. Ağırseven, M. A. Ollis, Standard and perfect linear realizations and the Buratti-Horak-Rosa Conjecture, *accepted.*

# Gregarious hypergraph designs

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A subhypergraph $\mathcal{H}$ of a given hypergraph is said to be gregarious with respect to a fixed vertex partition if the vertices of $\mathcal{H}$ belong to mutually distinct vertex classes. For graphs, this notion was introduced in the context of edge decomposition more than two decades ago, but its hypergraph generalization was first considered as late as in 2023, and only to the extent of just one theorem on 3-uniform hypergraphs. We begin the systematic study of gregarious decompositions of hypergraphs, with focus on complete $n$-partite $r$-uniform hypergraphs. Beyond their gregarious decompositions, a new approach is also offered and a related parameter introduced, expressing the gregarious decomposability of the blowups of a hypergraph.

## References

[1] E. J. Billington, D. G. Hoffman, *Decomposition of complete tripartite graphs into gregarious 4-cycles*, Discrete Math. **261** (2003), 87–111.

[2] P. Bonacini, L. Marino, Z. Tuza, *Gregarious Hypergraph Designs*, work in progress.

[3] Zs. Tuza, *Gregarious decompositions of complete equipartite graphs and related structures*, Symmetry **15**:12 (2023), paper 2097, 14 pp.

# Packing designs with large block size

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Given positive integers $v$, $k$, $t$ and $\lambda$ with $v \geq k \geq t$, a *packing design* $\mathrm{PD}_\lambda(v,k,t)$ is a pair $(V,\mathcal{B})$, where $V$ is a $v$-set and $\mathcal{B}$ is a collection of $k$-subsets of $V$ such that each $t$-subset of $V$ appears in at most $\lambda$ elements of $\mathcal{B}$. The maximum size of a $\mathrm{PD}_\lambda(v,k,t)$ is called the *packing number* and denoted $\mathrm{PDN}_\lambda(v,k,t)$.

We prove that for a positive integer $n$, $\mathrm{PDN}_\lambda(v,k,t) = n$ whenever $nk-(t-1)\binom{n}{\lambda+1} \leq \lambda v < (n+1)k-(t-1)\binom{n+1}{\lambda+1}$. For fixed $t$ and $\lambda$, this determines the value of $\mathrm{PDN}_\lambda(v,k,t)$ when $k$ is large with respect to $v$. By showing that if no point appears in more than three blocks, the blocks of a $\mathrm{PDN}_2(v,k,2)$ can be directed so that no ordered pair appears more than once, we also extend our results to directed packings with index $\lambda=1$ and strength $t=2$.

# Hamiltonian Cycles on Coverings

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A covering design is a $v$-set $V$ and a list $B$ of $b$ blocks of size $k$ where every pair from $V$ must occur in at least one block. A 1-block intersection graph (1-BIG) is a graph $G = (B, E)$, where $b \in B$ and $(b, b') \in E$ if $|b \cap b'| = 1$ for $b, b' \in B$. This talk will go over what independence sets look like in a 1-BIG based on coverings with $k = 3$. We prove that optimal $k = 3$ coverings $v \equiv 5 \pmod 3$ have a Hamiltonian cycle and show why this proof fails for even $v$ that are not Steiner Triple Systems.

## References

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# On the typical full automorphism group of Biembeddings of Archdeacon type

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In his seminal paper [1], Archdeacon introduced Heffter arrays as a tool to construct explicit $\mathbb{Z}_v$-regular biembeddings of complete graphs $K_v$ into orientable surfaces. The automorphism groups of these embeddings were later investigated in [3], where upper bounds on their size were established, and in [2], where it was shown that these bounds are attainable.

In this talk, we consider a generalization of the notion of Heffter array: the *quasi*-Heffter arrays. This framework yields 2-colorable Archdeacon-type embeddings of the complete multipartite graph $K_{\frac{v}{t}\times t}$ into orientable surfaces. We then show that the automorphism group of such embeddings is, in the generic case, precisely $\mathbb{Z}_v$.

## References

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# Switching operation for 2-designs and Hadamard matrices

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In this talk, we will present the switching operation for 2-designs introduced in [1]. This switching operation can be adapted for switching Hadamard matrices. Further, we show that this switching can be applied to any Bush-type Hadamard matrix.

## References

[1] D. Crnković, A. Švob, Switching for 2-designs, *Des. Codes Cryptogr.* 90 (2022), 1585–1593.

# Graham's rearrangement for a class of semidirect products

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A famous conjecture of Graham stated in 1971 asserts that for any set $A \subseteq \mathbb{Z}_p \setminus \{0\}$ there is an ordering $a_1, \ldots, a_{|A|}$ of the elements of $A$ such that the partial sums $a_1, a_1+a_2, \ldots, a_1+a_2+\ldots+a_{|A|}$ are all distinct. Before the recently improvements, the state of the art was essentially that the conjecture holds when $|A| \leq 12$ and when $A$ is a non-zero sum set of size $p-1$, $p-2$ or $p-3$. Many of the arguments for small $A$ use the Polynomial Method and rely on Alon's Combinatorial Nullstellensatz. Very recently, Kravitz in [3], using a rectification argument, made a significant progress proving that the conjecture holds whenever $|A| \leq \log p/\log\log p$. A subsequent paper of Bedert and Kravitz [1] improved the logarithmic bound into a super-logarithmic one that is of the form $e^{c(\log p)^{1/4}}$ for some small constant $c > 0$.

In [2], we use a similar procedure to obtain an upper bound of the same type in the case of semidirect products $\mathbb{Z}_p \rtimes_\varphi H$ where $\varphi : H \to Aut(\mathbb{Z}_p)$ satisfies $\varphi(h) \in \{id, -id\}$ for each $h \in H$ and where $H$ is abelian and each subset of $H$ can be ordered such that all of its partial products are distinct.

## References

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[3] N. Kravitz. Rearranging small sets for distinct partial sums, *Integers*, 24 (2024).

# Embedding partial Latin squares in Latin squares with many mutually orthogonal mates

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In this talk, I will review combinatorial constructions developed by Donovan, Grannell, and Yazıcı that verify that a pair of (partial) orthogonal Latin squares of order $n$, can be embedded in a set of $t+2$ mutually orthogonal Latin squares (MOLS) of polynomial order in $n$, for any $t \geq 2$. Notably, this construction verifies, for the first time, the existence of a set of nine MOLS of order 576, improving upon the earlier maximum of eight.

If time permits, I will also present earlier work by Donovan and Yazıcı, which provides the first constructive, polynomial-order embedding for a pair of orthogonal partial Latin squares.

# $G$-designs for some graphs on seven edges

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A *$G$-design of order $n$* is a collections of $s$ edge disjoint graphs $G_i$ isomorphic to $G$, whose union forms the complete graph $K_n$.

We complement a recent result by Fronček and Kubesa [2] by examining the remaining three disconnected bipartite graphs with seven edges: on nine and ten vertices. While the result itself is not too exciting, it provides an opportunity to present several different methods for finding $G$-designs. For $n \equiv 0, 1 \pmod{14}$, we use classical labeling methods introduced by Rosa [4] and generalized by El-Zanati et al. [3] and Bunge [1].

For $n \equiv 7 \pmod{14}$, we first decompose $K_{14k+7}$ into $K_{7,7}$ and $K_{14} - K_7$ and then each of them to $G$. For $n \equiv 8 \pmod{14}$, we decompose $K_{14k+8}$ into the circulant $Cir(14k+8; 1, 2, 7k+3, 7k+4)$ and its complement and then use labelings for the complement and labelings with some adjustments for the circulant.

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# Steiner triple systems with Veblen points

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A Steiner triple system, STS($v$), is a $2-(v, 3, 1)$ design. A *Veblen point* of an STS is a point for which any two other distinct points generate a Pasch configuration. Steiner triple systems given by the point-line design of a projective space PG($n, 2$) are precisely those in which every point is a Veblen point.

*Steiner loops* provide a natural algebraic framework for studying Steiner triple systems. We focus on their *Schreier extensions*, which offer an effective method for constructing Steiner triple systems with Veblen points. This concept was first introduced for loops in general in [1], and later explored in the context of Steiner loops in [2]. In particular, in [3] we investigate Veblen points in Steiner triple systems of orders 19, 27, and 31, determining their number and giving concrete examples.

## References

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# On decomposition thresholds for odd-length cycles

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An (edge) *decomposition* of a graph $G$ is a set of subgraphs of $G$ whose edge sets partition the edge set of $G$. I will discuss our recent proof that, for each odd $\ell \geq 5$, any graph $G$ of sufficiently large order $n$ with minimum degree at least $\big(\frac{1}{2} + \frac{1}{2\ell-4} + o(1)\big)n$ has a decomposition into $\ell$-cycles if and only if $\ell$ divides $|E(G)|$ and each vertex of $G$ has even degree. This threshold cannot be improved beyond $\frac{1}{2} + \frac{1}{2\ell-2}$. It was previously shown that the thresholds approach $\frac{1}{2}$ as $\ell$ becomes large, but our thresholds do so significantly more rapidly. Our methods can be applied to tripartite graphs more generally and we also obtain some bounds for decomposition thresholds of other tripartite graphs.

# On relative simple Heffter spaces

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Let G be an abelian group and suppose that $J$ is a subgroup of G of order $t$, a *half-set* $V$ of $G \backslash J$ is a subset of $G \backslash J$ such that for each non-involution element $x \in G \backslash J$, either $x$ or $-x$ is contained in $V$ and any involution elements of $G \backslash J$ are also contained in $V$. An $(nk, k)_t$ *relative Heffter system* is a partition of a half-set $V$ of $G \backslash J$ into zero-sum blocks of equal size. Two $(nk, k)_t$ relative Heffter systems $\mathcal{P}$ and $\mathcal{Q}$, based on the same half-set, are said to be *orthogonal* if their blocks intersect in at most one element. In a [1], we introduce the concept of a $(nk, k; r)_t$ *relative Heffter space*, which is a collection of $r$ mutually orthogonal Heffter systems. This definition naturally generalises the concepts of a relative Heffter array and a Heffter space.

The *density* of a Heffter space refers to the density of the collinear graph associated with the Heffter space. In the paper this talk is based on [1], we present two constructions of infinite families of relative Heffter spaces, that satisfy the additional property of being globally simple. One of these constructions always achieves maximal density. By obtaining results on globally simple Heffter spaces, [1] also obtains new results on mutually orthogonal cycle decompositions and biembeddings of cyclic cycle decompositions of the complete multipartite graph into an orientable surface.

## References

[1] L. Johnson, L. Mella, A. Pasotti, On relative simple Heffter spaces, arXiv preprint arXiv:2503.07445, (2025).

# On directed Oberwolfach problem with tables of even lengths

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A $(\vec{C}_{m_1}, \vec{C}_{m_2}, \ldots, \vec{C}_{m_t})$-factor of a directed graph $G$ is a spanning subdigraph of $G$ comprised of $t$ disjoint directed cycles of lengths $m_1, m_2, \ldots, m_t$, where $m_i \geqslant 2$. In this talk, we will be constructing a decomposition of the complete symmetric digraph $K_{2n}^*$ into $(\vec{C}_{m_1}, \vec{C}_{m_2}, \ldots, \vec{C}_{m_t})$-factors when $m_1 + m_2 + \cdots + m_t = 2n$, $t \geqslant 3$, and $n$ is odd. The existence of this decomposition implies a complete solution to the directed Oberwolfach problem with $t$ tables of even lengths and $2n$ guests such that $n$ is odd. This is joint work with Andrea Burgess and Peter Danziger.

# Further constructions of square integer relative Heffter arrays

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Heffter arrays are a fascinating combinatorial object introduced by Archdeacon in 2015 [1] and later generalized to *relative* Heffter arrays by Costa et al. in 2019 [2]. In this talk, I will focus on *square integer relative Heffter arrays*, which are $n \times n$ arrays where each row and column contains the same number of entries, the sum of each row and column is zero and where, given the subgroup $J$ of size $t$, every nonzero element of $\mathbb{Z}_{2nk+t} \setminus J$ appears exactly once up to sign. There are many open problems regarding the existence of these arrays. I will focus on arrays that contain *a primary transversal*, a transversal of the set $\{1, \ldots, n\}$ up to sign. I will present a new family of square integer relative Heffter arrays along with complete results for their existence for $n$ prime and $k = 3$[3].

## References

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# Balanced generalized kite designs

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In this work, we study valuations and labelings of bipartite graphs and their applications to cyclic graph designs. In particular, we introduce the notion of $(A, B)$-ordered and uniformly ordered labelings for a bipartite graph $G = (V, E)$ with partition classes $A$ and $B$. Using these labelings, we construct $(A, B)$-uniformly ordered labelings and describe how shifts of the labeling modulo $r + 1$ preserve certain ordering properties.

Our main result is a constructive method for cyclic $(C_m + P_{n+1})$-designs of order $v$, where $v \equiv 1 \mod 2(m + n)$. These results illustrate the interplay between ordered labelings of bipartite graphs and the construction of balanced generalized kite designs.

## References

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# Cycle decompositions of circulants $C(n, \{1, 3\})$

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A *decomposition* of a graph $G$ is a collection of edge-disjoint subgraphs $H_1, H_2, \ldots, H_t$ of $G$ such that each edge of $G$ belongs to exactly one $H_i$. We call this collection a *k-factorization* when every $H_i$ is a $k$-regular spanning subgraph of $G$.

For a positive integer $n$ and a set $S \subseteq \{1, \ldots, \lfloor(\frac{n}{2}\rfloor)\}$ a *circulant* $C(n, S)$ is a graph $G = (V, E)$ such that $V = \mathbb{Z}_n$ and $E = \{\{u, v\} : \delta(u, v) \in S\}$ where $\delta(u, v) = min\{\pm|u - v| \pmod{n}\}$.

Some results on decomposition of those graphs into cycles were obtained. Inspired by the work of Bryant and Martin [1], who gave a complete solution for the cycle decomposition of $C(n, \{1, 2\})$, we examine the case when $S = \{1, 3\}$. Among others, we present the results on decomposition of $C(n, \{1, 3\})$ into cycles of odd lengths and into cycles of even lengths.

In [2] Bryant showed that, whenever $n \geq 5$, there exists a 2-factorization of $C(n, \{1, 2\})$ in which one factor is a Hamiltonian cycle and the other factor is isomorphic to any given 2-regular graph of order $n$. We discuss some open problems concerning the 2-factorization of $C(n, \{1, 3\})$.

## References

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# Constructing magic objects

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In this talk I will describe some techniques that can be used to construct different magic objects such as signed magic arrays $\mathrm{SMA}(m, n; s, k)$, magic rectangle sets $\mathrm{MRS}(m, n; s, k; c)$ and integer Heffter arrays $\mathrm{H}(m, n; s, k)$. In the first two cases, we have determined necessary and sufficient conditions for their existence (see [1, 2]), while for integer Heffter arrays the problem is still open for some small values of $k$ (see [4]). Also, I will describe some constructions of $\Gamma$-magic rectangle sets $\mathrm{MRS}_\Gamma(m, n; s, k; c)$, where $\Gamma$ is a finite abelian group (see [3]).

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# On large Sidon sets

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A (binary) Sidon set $M$ is a subset of $\mathbb{F}_2^t$ such that the sum of four distinct elements of $M$ is never 0. The goal is to find Sidon sets of large size. In this talk we show that the graphs of almost perfect nonlinear (APN) functions with high linearity can be used to construct large Sidon sets. Thanks to recently constructed APN functions [[1, 2]]$\mathbb{F}_2^8 \to \mathbb{F}_2^8$ with high linearity, we can construct Sidon sets of size 192 in $\mathbb{F}_2^{15}$, where the largest sets so far had size 152. This result will be published in the Journal of Combinatorial Theory (A).

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# Exploring the Oberwolfach problem through solutions with non trivial automorphism group

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The Oberwolfach Problem, originally posed by Ringel in 1967, asks for a decomposition of complete graphs into 2-factors of prescribed isomorphism type. A recent asymptotic result (2021, Glock et al.) guarantees the existence of solutions provided that the total number of vertices is sufficiently large, yet, the possibility of determining explicit and constructible solutions for every configuration remains an open problem. Searching for solutions with a non trivial automorphism group may aid in constructing explicit examples, both in the classical setting and in variants obtained by adding or removing a repeatd $1-$factor from the complete graph.

# Distinct Difference Configurations

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A subset $D$ of a group is a Distinct Difference Configuration (DDC) if the differences $g^{-1}h$ are distinct, where $g$ and $h$ range over all (ordered) pairs of distinct elements of $D$. When developed over the group, the resulting blocks form a $2-(|G|,|D|,1)$ packing.

DDCs also have practical applications in key predistribution schemes for wireless sensor networks, especially when the network's communication structure mirrors the Cayley graph of a group. In such a scheme, using a bounded DDC ensures an efficient trade-off between local connectivity and global security.

This talk will also cover joint work with Luke Stewart and Simon Blackburn where we show that large, bounded DDCs exist in free groups, extending the theory beyond abelian or finite settings.

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# On some new regular digraphs from finite groups

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In this talk, we describe a construction of certain regular digraphs using finite simple groups. We introduce the notion of orbit matrices of digraphs and point out some interesting results obtained using specific linear groups. In particular, we present the first example of a directed strongly regular graph with parameters (63,11,8,1,2) along with several other new directed strongly regular graphs obtained from finite simple groups. The talk is based on the papers [1, 2, 3].

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# An explicit lower bound on the largest cycle for the solvability of the Oberwolfach problem

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The Oberwolfach problem OP($F$), posed by Ringel in 1967, asks for a decomposition of the complete graph $K_v$ into copies of a given 2-regular graph $F$ of odd order $v$. Some recent non-constructive results [2, 4] provide an asymptotic proof of the solvability of OP($F$) for sufficiently large orders, but leave the specific lower bound for $v$ unquantified.

In this talk, we present a method to build solutions to OP($F$) whenever $F$ has a cycle of length greater than an explicit lower bound [5], thereby partially filling this gap. Our constructions combine the amalgamation-detachment technique [3] with methods for building suitable decompositions of $K_v$ having an automorphism group with a nearly-regular action on the vertices [1].

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