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Book of abstracts

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List of talks

On the optimality criteria of tree decompositions

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Tree decomposition is an important tool used in algorithmic and structural graph theory. Intuitively, a tree decomposition represents the vertices of a graph G as subtrees of some tree T, in such a way that vertices in G are adjacent only when the corresponding subtrees intersect. On the other hand — vertices of T may be viewed as collections of subtrees (and corresponding vertices of G) and thus they are called *bags*.

Given graph G a natural question is how to optimally choose a tree T with the particular decomposition. The standard approach is to have the largest bag as small as possible, which leads to the notion of *tree-width*. Recently, another optimization criteria were considered (eg. *tree independence number*, where the largest size of the maximum independent set of the subgraph of G induced by any bag is optimized).

We discuss some of these criteria, in particular we show the connection between *tree domination number* and a *hypertree-width* (the standard measure of tree decomposition of hypergraphs).

Strong chordality in digraphs

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I will discuss digraph analogues of well structured graph classes such as interval graphs and chordal graphs, with emphasis on new results on strong chordality in digraphs. These are joint work with Cesar Hernandez Cruz, and Jing Huang; the older work involves also collaborations with Sandip Das, Tomás Feder, Mathew Francis, Jephian Lin, Ross McConnell, and Arash Rafiey.

K-Coloring $(bull, chair)$-free graphs

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The k-Coloring problem is NP-hard in general, but it becomes tractable in some hereditary graph classes. We show that it can be solved in polynomial time for $(bull, chair)$-free graphs. Here, *chair* is a 3-star $S_{1,1,2}$ with one edge subdivided and bull is a triangle with two additional leaves attached to two vertices.

The algorithm we present in this talk resolves even a more general List k-Coloring problem: given a graph G and a set of lists $\{L(v) : v \in V(G), L(v) \subset [k]\}$, we look for a proper coloring c of $V(G)$ such that $c(v) \in L(v)$ for every vertex v. The algorithm works recursively, where base trivial case is $|V(G)| = 1$ or $\max_{v \in V(G)} |L(v)| = 1$. In one step we exhaustively guess the coloring of an expansion of path $R \subset G$ and for each coloring guessed we adjust the lists and call the algorithm on the components of $G - R$. In each descendant call maximum length of the lists decreases, so the depth of recursion is bounded by k.

References

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[2] P. Gartland, D. Lokshtanov, T. Masařík, M. Pilipczuk, M. Pilipczuk, and P. Rzążewski. *Maximum weight independent set in graphs with no long claws in quasi-polynomial time.* In B. Mohar, I. Shinkar, and R. O'Donnell, editors, *Proceedings of the 56th Annual ACM Symposium on Theory of Computing, STOC 2024*, Vancouver, BC, Canada, June 24-28, 2024, pages 683–691. ACM, 2024.

Coloring Mixed Graphs

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A *mixed graph* is defined as a triple (V, E, A), where V is the set of vertices, E is the set of undirected edges, and A is the set of directed edges (arcs). A proper coloring of a mixed graph is an assignment of positive integers (colors) to the vertices such that adjacent vertices connected by an undirected edge receive distinct colors, and for every directed edge $(u, v) \in A$, the color assigned to vertex v is strictly greater than the color assigned to u.

In this talk, we present two algorithms for computing the minimal number of colors required to properly color a mixed graph. The first algorithm operates in exponential space, while the second in polynomial space. We further analyze the computational complexity of these algorithms, revealing connections to several interesting problems in extremal graph theory.

Rendezvous of heterogeneous agents and multimode graphs

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In the rendezvous problem, two mobile agents move along edges from node to node, with the goal of occupying the same node at the same time. Once, the time required to move along the edge is defined separately for each agent, we call them heterogeneous [1].

Intuitively, the very large weight assigned to an edge makes it practically inaccessible to the agent. This leads to a concept in which the graph G is given with two sets of edges E_A and E_B that define availability zones for A and B agents, respectively [2].

The very similar concept with separate edge sets that define availability zones was used to analyze evacuation and searching problems in graphs. In the independent studies, the same object is called a *multimode graph* [3] with the motivations coming from multi 'mode' transport where different 'modes' are not combinable, such as flights operated by different airline alliances.

References

[1] D.Dereniowski, R.Klasing, A.Kosowski, ŁKuszner, Rendezvous of heterogeneous mobile agents in edge-weighted networks. SIROCCO 2014 pp.311-326.

[2] A.Farrugia, L.Gasieniec, Ł.Kuszner, E.Pacheco, Deterministic Rendezvous in Restricted Graphs, SOFSEM 2015 pp.189-200.

[3] Y.Kirkpatrick, V.Vassilevska Williams, Shortest Paths in Multimode Graphs, MFCS 2025 pp. 63:1-63:16.

Linear colorings of graphs

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Motivated by algorithmic applications, Kun, O'Brien, Pilipczuk, and Sullivan [2] introduced the parameter linear chromatic number as a relaxation of treedepth and proved that the two parameters are polynomially related. They conjectured that treedepth could be bounded from above by twice the linear chromatic number. We investigate the properties of linear chromatic number and provide improved bounds in several graph classes (see [1]).

References

[1] C. Hilaire, M. Krnc, M. Milanič, J.-F. Raymond. Linear colorings of graphs. Preprint, 2025. `https://arxiv.org/abs/2505.02768`.

[2] J. Kun, M. P. O'Brien, M. Pilipczuk, and B. D. Sullivan, Polynomial treedepth bounds in linear colorings, *Algorithmica* 83 (2021) 361–386.

Layered tree-independence number and clique-based separators

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Motivated by a question of Galby, Munaro, and Yang (SoCG 2023) asking whether every graph class of bounded layered tree-independence number admits clique-based separators of sublinear weight, we investigate relations between layered tree-independence number, weight of clique-based separators, clique cover degeneracy and independence degeneracy. In particular, we provide a number of results bounding these parameters on geometric intersection graphs. For example, we show that the layered tree-independence number is $\mathcal{O}(g)$ for g-map graphs, $\mathcal{O}(\frac{r}{\tanh r})$ for hyperbolic uniform disk graphs with radius r, and $\mathcal{O}(1)$ for spherical uniform disk graphs with radius r. Our structural results have algorithmic consequences. In particular, we obtain a number of subexponential or quasi-polynomial-time algorithms for weighted problems such as Max Weight Independent Set and Min Weight Feedback Vertex Set on several geometric intersection graphs. Finally, we conjecture that every fractionally tree-independence-number-fragile graph class has bounded independence degeneracy.

Hitting all longest paths in hereditary graph classes

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The *longest path transversal number* of a connected graph G, is the minimum size of a set of vertices of G hitting all longest paths in G. Surprisingly, it is not known whether there exists a constant upper bound for the longest path transversal number of connected graphs. We present constant upper bounds for the longest path transversal number of multiple *hereditary classes of graphs*, that is, classes of graphs which are closed under induced subgraph containment, including the class of P_6-free graphs.
Based on a joint work with Paloma T. Lima and Paweł Rzążewski.

A Faster Algorithm for Independent Cut

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The previously fastest algorithm for deciding the existence of an independent cut had a runtime of $O^*(1.4423^n)$, where n is the order of the input graph. We improve this to $O^*(1.4143^n)$. In fact, we prove a runtime of $O^*\left(2^{(\frac{1}{2}-\alpha_\Delta)n}\right)$ on graphs of order n and maximum degree at most Δ, where $\alpha_\Delta = \frac{1}{2+4\lfloor\frac{\Delta}{2}\rfloor}$. Furthermore, we show that the problem is fixed-parameter tractable on graphs of order n and minimum degree at least βn for some $\beta > \frac{1}{2}$, where β is the parameter.

MIS on graphs excluding induced substructures

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The complexity of the Maximum Independent Set problem is fully classified for graph classes defined by forbidden subgraphs or minors. It is polynomial-time solvable when excluding a forest in which every tree has at most three leaves as a subgraph, or when excluding a planar graph as a minor; in all other cases, it remains NP-hard. For graph classes defined by forbidden induced subgraphs or induced minors, however, the complexity landscape is largely unresolved. This talk presents some of the resolved cases ([1], [2]) and their connections to related problems.

References

[1] É. Bonnet, J. Duron, C. Geniet, S. Thomassé, A. Wesolek, Maximum Independent Set when excluding an induced minor: $K_1 + tK_2$ and $tC_3 \uplus C_4$, *ESA*, 2023 pp. 23:1–23:15.

[2] M. Bonamy, É. Bonnet, H. Déprés, L. Esperet, C. Geniet, C. Hilaire, S. Thomassé, A. Wesolek, Sparse graphs with bounded induced cycle packing number have logarithmic treewidth, *J. Comb. Theory Ser. B*, 2024 pp. 215-249.

Algorithmically on vertices that belong to all, some and no minimum dominating set in a tree

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A subset D of V_G is said to be a *dominating set* of a graph $G = (V_G, E_G)$ if each vertex in the set $V_G \setminus D$ has a neighbour in D. The (*independent*) *domination number* of G, denoted by $\gamma(G)$ (resp., by $\gamma_i(G)$), is defined to be the minimum cardinality of a (independent) dominating set D of G, and any minimum (independent) dominating set of G is referred to as a γ-set (resp., as a γ_i-set).

We propose a linear time algorithm for determining the sets of vertices that belong to all, some and no minimum dominating set of a tree, respectively, thus improving the quadratic time algorithm of Benecke and Mynhardt in 2008 [S. Benecke, C.M. Mynhardt, Trees with domination subdivision number one, *Australasian Journal of Combinatorics* 42, 201-209 (2008)].

Our result immediately implies the following corollaries: *For any tree T, the following problems are solvable in linear time and space: (A) The problem of verifying whether T is γ-excellent; (B) The problem of verifying whether* $\mathrm{sd}_\gamma(T) = 1$*; (C) The problem of verifying whether* $\mathrm{sd}_{\gamma_i}(T) = 1$. Recall that for a given graph parameter μ, the μ-subdivision number sd_μ is defined to be the minimum number of edges that must be subdivided to change μ, where each edge may be subdivided at most once.