10th Cracow Conference on Graph Theory

Book of abstracts

Cracow, September 22-26, 2025

https://10ccgt.agh.edu.pl/

Keynote speakers

Thematic sessions

Graph-Codes: Problems, Results and Methods

Noga Alon[1,2]

[1] Princeton University, Princeton, USA
[2] Tel Aviv University, Tel Aviv, Israel

The study of Graph-Codes is motivated by questions in Extremal Combinatorics, Additive Number Theory and Coding Theory. The initial guiding fact is that viewing binary vectors as characteristic vectors of edge-sets of graphs transforms the basic combinatorial questions of Coding Theory into intriguing extremal problems about families of graphs. I will discuss some of these questions and describe several results and open problems. The relevant methods combine Combinatorial and Probabilistic tools with techniques from Information Theory, Number Theory and the theory of Combinatorial Designs.

Vertex sets in a hypercube: fair distribution and quick secret sharing

Maria Axenovich[1]

[1] Karlsruhe Institute of Technology, Karlsruhe, Germany

Hypercubes are highly symmetrical graphs important for their mathematical properties, such as representing set systems, as well as for their applications in computer science and data analysis due to efficient communication pathways.

This talk will address two questions about special vertex sets in a hypercube Q_n:

For what parameters (n, d, s) is there a set A of vertices in Q_n that is distributed "fairly", i.e., such that each sub-hypercube of dimension d has exactly s vertices from A?

What is the largest size of a set B of vertices in Q_n that can share secrets quickly, i.e., such that for any two vertices from B there is a shortest in Q_n path between them that contains no other vertices from B.

The talk is based on a joint work with Noga Alon, John Goldwasser, and Dingyuan Liu.

On fractional chromatic number and its approximations

Zdeněk Dvořák(1)

(1) Charles University, Prague, Czechia

In fractional coloring, we color vertices of a graph by sets of measure one subject to the constraint that adjacent vertices receive disjoint sets, and we minimize the measure of the union of these sets. This is a natural generalization of the usual graph coloring, which can be viewed as a special case when the measure space is finite. Hence, fractional chromatic number can be used as a lower bound for the ordinary chromatic number. Moreover, optimal fractional coloring reveals interesting information about independent sets of the graph.

Unsurprisingly, it is hard (both from the theoretical and the computational perspective) to determine the fractional chromatic number exactly. Thus, it is natural to ask whether we can approximate it or bound it in terms of simpler graph parameters. In my talk, I will discuss recent results concerning several interpretations of this question.

Configuration space of 2-cell embeddings of graphs in surfaces and the Genus Log-Concavity Conjecture

Bojan Mohar[1,2]

[1] Simon Fraser University, Burnaby, Canada
[2] University of Ljubljana, Ljubljana, Slovenia

Every 2-cell embedding of a graph in some closed surface can be described (canonically) by specifying local clockwise rotations of edges emanating from each vertex. The set of all such rotation systems on a given graph can be viewed as a large graph, called the configuration space of 2-cell embeddings of the graph. With special emphasis on cubic graphs, the talk will discuss how many embeddings of certain genus can we have. An application towards and against the Genus Log-Concavity Conjecture will be presented. The main part of the talk is joint work with MacKenzie Carr.

Degree-truncated choice number of graphs

X. Zhu(1)

(1) Zhejiang Normal University, Jinhua, China

Assume G is a connected graph and k is a positive integer. We say G is degree-truncated k-choosable if G is f-choosable, where $f(v) = \min\{d_G(v), k\}$. The degree-truncated choice number $ch^*(G)$ of G is the minimum k such that G is degree-truncated k-choosable. For a family $\mathcal{G}$ of graphs, let $ch^*(\mathcal{G}) = \max\{ch^*(G) : G \in \mathcal{G}\}$. If G is a Gallai-tree, then G is not degree-choosable, and hence G is not degree-truncated k-choosable for any integer k. In this case, the degree-truncated choice number of G is not defined. Otherwise, $ch^*(G)$ is well-defined and is at most the maximum degree of G.

In this talk, I will survey recent progress on the study of degree-truncated choice number of graphs, and pose some questions. This talk includes results in a few papers joint with C. Deng, Y. Jiang, S. Lo, C. Wang, H. Xu, X. Xu, H. Zhou and J. Zhu.

References

[1] C. Deng and X. Zhu, Degree-truncated Alon-Tarsi number of outerplanar graphs, *Manuscript*, 2025.

[2] J. Hutchinson, On list-Coloring outerplanar graphs, *J. Graph Theory* 59 (2008), 59-74.

[3] Y. Jiang, H. Xu, X. Xu and X. Zhu, Degree-truncated choosability of planar graphs, *Manuscript*, 2025.

[4] O. S. Lo, C. Wang, H. Zhou and X. Zhu, $K_{2,4}$-minor free graphs are DP-5-truncated degree-colourable, *arXiv:2312.15962.*

[5] H. Zhou, J. Zhu and X. Zhu, Degree-truncated choosability of graphs, *Journal of Combinatorial Theory Ser. B*, to appear (2025).

Connections between local regularity and symmetry in graphs

Robert Jajcay[(1)]

[(1)] Comenius University, Bratislava, Slovakia

When talking about highly symmetric graphs, we may be talking about several different but interconnected concepts. We may be talking about graphs with large automorphism groups (in comparison to their orders), or graphs whose automorphism groups act in a special way (vertex-transitive, edge-transitive, arc-transitive, etc.), or we can even be talking about graphs that appear symmetric (but may not necessarily be), by which we may mean some kind of a regularity shared by graphs from the other two classes defined by symmetries viewed as automorphisms.

In our presentation, we aim to connect these various concepts of symmetry via discussing hierarchies within the above classes of graphs and the links connecting the various levels of these hierarchies. We will also discuss possibilities for loosening or generalizing some of the discussed concepts in order to explore connections to even wider classes defined via the existence or possibly even lack of specific symmetries. Some of the topics discussed will include smallest vertex-transitive subgroups and families of automorphisms and their connection to Cayley and quasi-Cayley graphs, the monoid of partial automorphisms of a graph, and girth-regular, edge-girth-regular, and vertex-girth-regular graphs.

Parts of this talk may also be viewed as an introduction to presentations included in the *Algebraic Graph Theory* session of the 10th Cracow Conference 2025.

Uniform routings of shortest paths in graphs with large automorphism groups

V. Cingel[(1)], R. Jajcay[(1,2)]

[(1)] Comenius University, Bratislava, Slovakia
[(2)] Supervisor

A *routing* R of a given connected graph Γ of order n is a collection of $n(n-1)$ paths connecting every ordered pair of distinct vertices of Γ. If these paths are the shortest, R is called a *routing of shortest paths.* The *load* $\xi(\Gamma, R, v)$ of a given vertex v is the number of paths of R passing (not beginning or ending) through v. When designing networks, it is often beneficial when every vertex has the same load. Such a routing is called a *uniform routing of shortest paths*, or shortly URSP.

Many vertex-transitive (v-t) graphs have been shown to admit a URSP. For example, all Cayley and quasi-Cayley graphs, i.e. all graphs containing a regular set of automorphisms [1]. However, not all v-t graphs belong in this class - the underlying graph of the dodecahedron does not admit a URSP [2]. The connection between the level of 'symmetry' of a graph and the existence of a URSP is not well understood. We have shown that vertex-transitivity is not necessary for the existence of a URSP. However, in the case of geodetic graphs, i.e. graphs having precisely one shortest path between any two vertices, we have shown that vertex-transitivity suffices. We will present a classification of planar geodetic graphs with respect to the existence of URSP.

References

[1] G.Gauyacq, On quasi-Cayley graphs, *Discrete Appl. Math.* No. 1, 1997 pp.43-58.

[2] S.Shim, J.Širáň, and J.Žerovnik, Counterexamples to the uniform shortest path routing conjecture for vertex-transitive graphs. *Discrete Appl. Math.* 119.3, 2002, pp.281–286.

Cubic girth-regular graphs of girth six

Š. Glevitzká[(1)], R. Jajcay[(1)], M. Lekše[(2)], P. Potočnik[(3)]

[(1)] Comenius University, Bratislava, Slovakia
[(2)] IMFM, Ljubljana, Slovenia
[(3)] University of Ljubljana, Ljubljana, Slovenia

Recall that given a graph Γ, the girth of Γ is the length of a shortest cycle in Γ. If the girth is finite, then each vertex v of Γ can be associated with a list of integers, one integer for each edge incident with v, representing the number of girth cycles containing the edge (possibly 0). If the lists associated with the vertices of Γ are all identical (and so Γ is regular), we say that Γ is *girth-regular*, and the shared list is said to be the *signature of* Γ. Note that all vertex-transitive graphs are necessarily girth-regular.

The concept of girth-regularity was introduced in [1], where several necessary conditions on signatures of cubic girth-regular graphs were proved, together with a classification of all cubic girth-regular graphs of girth at most 5. Consequently, all cubic vertex-transitive graphs of girth 6 were classified in [2]. In our work, we extend the latter to a characterization of all cubic girth-regular graphs of girth 6. In addition, we prove multiple additional conditions on signatures of cubic girth-regular graphs of any even girth.

The work has been supported by grants VEGA 1/0437/23 and UK/1398/2025.

References

[1] P.Potočnik, J.Vidali, Girth-regular graphs, *Ars Math. Contemp.* 2019 pp.349–368.

[2] P.Potočnik, J.Vidali, Cubic vertex-transitive graphs of girth six, *Discrete Math.* 2022.

Prime factorization of hierarchical products of infinite graphs

Wilfried Imrich[1]

[1] Department Mathematics and Information Technology, Montanuniversität Leoben, A-8700 Leoben, Austria

All connected finite graphs have unique prime factorization with respect to the hierarchical product, as was shown in a joint paper with Kalinowski and Pilśniak. But this does not extend to connected infinite graphs. Here it is shown that all connected infinite rayless graphs have unique prime factorization with respect to the hierarchical product, and conditions are provided under which this also holds for locally finite infinite graphs. Furthermore, factorization properties of special classes of graphs are investigated, as well as the structure of the automorphism groups of hierarchical products.

Asymmetric depth of graphs

Tatiana Jajcayova(1)

(1) Comenius University, Bratislava, Slovakia

A partial automorphism of a graph is an isomorphism between its induced subgraphs. The set of all partial automorphisms of a given graph forms an inverse monoid under composition of partial maps and taking partial inverses. In constrast to classical group theory approach to studying symmetries of graphs, where the automorphism group of a graph can be (and almost always is) trivial, the inverse monoid of partial automorphisms of a graph is never trivial. It gives full algebraic description of the graph [1]. In our talk, motivated by the study of partial automorphism inverse monoids of graphs, we investigate symmetries of graphs utilising the measure of asymmetric depth of graphs defined through the rank of the largest nontrivial partial automorphism. We establish a new, tight lower bound for the asymmetric depth of any simple graph Γ of order n. Any graph achieving this bound must be a strongly regular graph with parameters $(n, \frac{n-1}{2}, \frac{n-5}{4}, \frac{n-1}{4})$ also known as *conference graph*. Via computation performed on a high-performance cluster, we were able to identify an asymmetric conference graph on 37 vertices that meets this bound, thereby proving its tightness. We also show that it is one of the smallest possible graphs to meet this bound.

This is joint work with Ján Pastorek.

Acknowledgments: The work has been supported by APVV grants SK-AT-23-0019, APVV-23-0076, and by VEGA grant 1/0437/23

References

[1] R. Jajcay, T. Jajcayova, N. Szakács, and M.B. Szendrei, Inverse monoids of partial graph automorphisms, *Journal of Algebraic Combinatorics,* 2021 vol. 53, no. 3, pp.829–849.

How can graphs help us count?

Pavol Kollár(1)

(1) Comenius University, Bratislava, Slovakia

This talk is about the topic of *Boundary Matrices*, which are matrices specified by a set of forbidden submatrices that these Boundary Matrices must avoid containing. In this sense, these matrices are a generalisation of the antidictionary languages, which is a classical - solved - problem [1]. Ideas similar to boundary matrices show up in various areas of research [2], [3].

In addition to asking "how many boundary matrices without forbidden submatrices $Z_1, \ldots, Z_x$ are there of size $m \times n$?", our focus of the project is to examine the relation between the numbers of these matrices. The main interest is answering a conjecture from [4], whether there exists a two-dimensional linear recurrence relation between these numbers of boundary matrices.

As we will see, the adjacency matrix of a directed graph related to building these boundary matrices plays a key role in determining the answer to the conjecture.

I acknowledge Funding by the EU NextGenerationEU through the Recovery and Resilience Plan for Slovakia under the project No. 09I03-03-V02-00036. I also acknowledge support from the Grant of Comenius University No. UK/1188/2025.

References

[1] Alfred V. Aho, Margaret J. Corasick, Efficient string matching: an aid to bibliographic search, *Commun ACM.* (1975) 333–340.

[2] Silvère Gangloff, Mathieu Sablik, Quantified block gluing for multidimensional subshifts of finite type: aperiodicity and entropy. *Journal d'Analyse Mathématique* (2021) 21–118.

[3] Elliott H. Lieb, Exact Solution of the Problem of the Entropy of Two-Dimensional Ice. *Physical Review Letters* (1967) 692–694.

[4] Robert Jajcay, Tatiana Jajcayova, Marian Opial, Enumeration of matrices with prohibited bounded sub-windows. *Conference Information Technologies – Applications and Theory 2018* 176–180.

Cubic vertex-transitive graphs of girth seven

M. Lekše[1,2], P. Potočnik[1,2], M. Toledo[1,2]

[1] University of Ljubljana, Ljubljana, Slovenia
[2] IMFM, Ljubljana, Slovenia

Let Γ be a graph, and let v be a vertex and e an edge of Γ. The signature of e is the number of girth cycles that contain it, while the signature of v is the tuple of the signatures of all edges incident to it (ordered by size). We say that Γ is girth-regular if every vertex in the graph has the same signature. This concept was introduced by Potočnik and Vidali in 2019 as a generalization of edge-girth-regularity, and they later used it to classify cubic vertex-transitive graphs of girth at most 6. In this talk, we present a similar classification of cubic vertex-transitive graphs of girth 7 based on their signatures.

On bipartite biregular large graphs derived from difference sets

G. Araujo-Pardo[(1)], C. Dalfó[(2)], M. A. Fiol[(3)], N. López[(2)]

[(1)] Universidad Nacional Autónoma de México, México.
[(2)] Universitat de Lleida, Spain.
[(3)] Universitat Politècnica de Catalunya, Barcelona, Spain.

A bipartite graph $G = (V, E)$ with $V = V_1 \cup V_2$ is biregular if all the vertices of each stable set, V_1 and V_2, have the same degree, r and s, respectively. This paper studies difference sets derived from both Abelian and non-Abelian groups. From them, we propose some constructions of bipartite biregular graphs with diameter $d = 3$ and asymptotically optimal order for given degrees r and s. Moreover, we find some biMoore graphs, that is, bipartite biregular graphs that attain the Moore bound.

References

[1] G. Araujo-Pardo, C. Dalfó, M. A. Fiol, and N. López, Bipartite biregular Moore graphs, *Discrete Math.* **344** (2021) 112582.

[2] G. Erskine, P. Fratrič, and J. Širáň, Graphs derived from perfect difference sets, *Australas. J. Combin.* ***80*** *(2021), no. 1, 48–56.*

[3] S. Filipovski, A. Ramos Rivera, and R. Jajcay, On biregular bipartite graphs of small excess, *Discrete Math.* **342** (2019) 2066–2076.

[4] J. L. A. Yebra, M. A. Fiol, and J. Fàbrega, Semiregular bipartite Moore graphs, *Ars Combin.* **16A** (1983) 131–139.

Cubic vertex-transitive graphs with a symmetry-invariant 2-factor – a generalisation of the Generalised Petersen graphs

P. Šparl(1)

(1) University of Ljubljana, Ljubljana, Slovenia

We present a few results obtained during the investigations spurred by the question of Bojan Mohar to Brian Alspach from 2016 on cubic vertex-transitive graphs admitting a 2-factor which is invariant under the full automorphism group of the graph. It is not difficult to show that the graphs admitting such a 2-factor are precisely the cubic vertex-transitive but not arc-transitive graphs, but obtaining a precise description of the graphs in terms of the corresponding 2-factor and the complementing 1-factor presents an interesting challenge.

While the simplest case when the 2-factor is a Hamiltonian cycle of the graph was settled back in 2019 by Alspach, Khodadadpour and Kreher, the author of this talk (and Ted Dobson) became involved in the investigations when the case that the 2-factor consists of two cycles was considered. The main purpose of this talk is to present some more recent results [1], where the situation that the quotient with respect to the 2-factor is a cycle, was investigated thoroughly. The resulting graphs in one of the two essentially different cases represent a very natural generalisation of the Generalised Petersen graphs (as well as of the perhaps somewhat lesser-known Honeycomb toroidal graphs).

Some of the presented results are joint work with Brian Alspach.

References

[1] B. Alspach, P. Šparl, Cubic factor-invariant graphs of cycle quotient type – The alternating case, *European J. Combin.* 120 (2024), Paper No. 103964, 22 pp.

Networks with small excess

J. Tuite(1,2)

(1) Open University, Milton Keynes, UK
(2) Klaipėda University, Klaipėda, Lithuania

The *degree/geodecity problem* is an analogue for directed and mixed graphs of the classical degree/girth problem [1]. It asks for the smallest networks with fixed out-degree (as well as undirected degree for mixed graphs) such that for any two vertices u, v there is at most one (non-backtracking) walk from u to v with length at most k. The directed/mixed Moore bound is a lower bound for the problem. We present new results on the structure of networks with order close to the Moore bound, including their regularity properties, and a new lower bound for mixed graphs for $k = 2$.

References

[1] J. Tuite, G. Erskine, On networks with order close to the Moore bound, *Graphs Comb.* 38 (5) (2022) 143.

On the optimality criteria of tree decompositions

Aleksandra Czerczak[1], Przemysław Gordinowicz[1]

[1] Lodz University of Technology, Łódź, Poland

Tree decomposition is an important tool used in algorithmic and structural graph theory. Intuitively, a tree decomposition represents the vertices of a graph G as subtrees of some tree T, in such a way that vertices in G are adjacent only when the corresponding subtrees intersect. On the other hand — vertices of T may be viewed as collections of subtrees (and corresponding vertices of G) and thus they are called *bags*.

Given graph G a natural question is how to optimally choose a tree T with the particular decomposition. The standard approach is to have the largest bag as small as possible, which leads to the notion of *tree-width*. Recently, another optimization criteria were considered (eg. *tree independence number*, where the largest size of the maximum independent set of the subgraph of G induced by any bag is optimized).

We discuss some of these criteria, in particular we show the connection between *tree domination number* and a *hypertree-width* (the standard measure of tree decomposition of hypergraphs).

Strong chordality in digraphs

Pavol Hell[1]

[1] Simon Fraser University, Burnaby, B.C.

I will discuss digraph analogues of well structured graph classes such as interval graphs and chordal graphs, with emphasis on new results on strong chordality in digraphs. These are joint work with Cesar Hernandez Cruz, and Jing Huang; the older work involves also collaborations with Sandip Das, Tomás Feder, Mathew Francis, Jephian Lin, Ross McConnell, and Arash Rafiey.

K-Coloring $(bull, chair)$-free graphs

N. Hodur[(1)], M. Pilśniak[(1)], M. Prorok[(1)], P. Rzążewski[(2)]

[(1)] AGH University of Krakow, Poland

[(2)] Warsaw University of Technology & University of Warsaw, Poland

The k-Coloring problem is NP-hard in general, but it becomes tractable in some hereditary graph classes. We show that it can be solved in polynomial time for $(bull, chair)$-free graphs. Here, *chair* is a 3-star $S_{1,1,2}$ with one edge subdivided and bull is a triangle with two additional leaves attached to two vertices.

The algorithm we present in this talk resolves even a more general List k-Coloring problem: given a graph G and a set of lists $\{L(v) : v \in V(G), L(v) \subset [k]\}$, we look for a proper coloring c of $V(G)$ such that $c(v) \in L(v)$ for every vertex v. The algorithm works recursively, where base trivial case is $|V(G)| = 1$ or $\max_{v \in V(G)} |L(v)| = 1$. In one step we exhaustively guess the coloring of an expansion of path $R \subset G$ and for each coloring guessed we adjust the lists and call the algorithm on the components of $G - R$. In each descendant call maximum length of the lists decreases, so the depth of recursion is bounded by k.

References

[1] N. Hodur, M. Pilśniak, M. Prorok, and I. Schiermeyer. *On 3-colourability of $(bull, H)$-free graphs.* Preprint available at `https://arxiv.org/abs/2404.12515`, 2024.

[2] P. Gartland, D. Lokshtanov, T. Masařík, M. Pilipczuk, M. Pilipczuk, and P. Rzążewski. *Maximum weight independent set in graphs with no long claws in quasi-polynomial time.* In B. Mohar, I. Shinkar, and R. O'Donnell, editors, *Proceedings of the 56th Annual ACM Symposium on Theory of Computing, STOC 2024*, Vancouver, BC, Canada, June 24-28, 2024, pages 683–691. ACM, 2024.

Coloring Mixed Graphs

Grzegorz Gutowski[1], Konstanty Junosza-Szaniawski[2], Antonio Lauerbach[3], Paweł Rzążewski[2], Alexander Wolff[3]

[1] Jagiellonian University
[2] Warsaw University of Technology
[3] University of Würzburg

A *mixed graph* is defined as a triple (V, E, A), where V is the set of vertices, E is the set of undirected edges, and A is the set of directed edges (arcs). A proper coloring of a mixed graph is an assignment of positive integers (colors) to the vertices such that adjacent vertices connected by an undirected edge receive distinct colors, and for every directed edge $(u, v) \in A$, the color assigned to vertex v is strictly greater than the color assigned to u.

In this talk, we present two algorithms for computing the minimal number of colors required to properly color a mixed graph. The first algorithm operates in exponential space, while the second in polynomial space. We further analyze the computational complexity of these algorithms, revealing connections to several interesting problems in extremal graph theory.

Rendezvous of heterogeneous agents and multimode graphs

Łukasz Kuszner[(1)]

[(1)] University of Gdansk, Poland

In the rendezvous problem, two mobile agents move along edges from node to node, with the goal of occupying the same node at the same time. Once, the time required to move along the edge is defined separately for each agent, we call them heterogeneous [1].

Intuitively, the very large weight assigned to an edge makes it practically inaccessible to the agent. This leads to a concept in which the graph G is given with two sets of edges E_A and E_B that define availability zones for A and B agents, respectively [2].

The very similar concept with separate edge sets that define availability zones was used to analyze evacuation and searching problems in graphs. In the independent studies, the same object is called a *multimode graph* [3] with the motivations coming from multi 'mode' transport where different 'modes' are not combinable, such as flights operated by different airline alliances.

References

[1] D.Dereniowski, R.Klasing, A.Kosowski, ŁKuszner, Rendezvous of heterogeneous mobile agents in edge-weighted networks. SIROCCO 2014 pp.311-326.

[2] A.Farrugia, L.Gasieniec, Ł.Kuszner, E.Pacheco, Deterministic Rendezvous in Restricted Graphs, SOFSEM 2015 pp.189-200.

[3] Y.Kirkpatrick, V.Vassilevska Williams, Shortest Paths in Multimode Graphs, MFCS 2025 pp. 63:1-63:16.

Linear colorings of graphs

C. Hilaire(1), M. Krnc(1), M. Milanič(1), J.-F. Raymond(2)

(1) FAMNIT and IAM, University of Primorska, Koper, Slovenia
(2) CNRS, ENS de Lyon, Université Claude Bernard Lyon 1, LIP UMR 5668, Lyon, France

Motivated by algorithmic applications, Kun, O'Brien, Pilipczuk, and Sullivan [2] introduced the parameter linear chromatic number as a relaxation of treedepth and proved that the two parameters are polynomially related. They conjectured that treedepth could be bounded from above by twice the linear chromatic number. We investigate the properties of linear chromatic number and provide improved bounds in several graph classes (see [1]).

References

[1] C. Hilaire, M. Krnc, M. Milanič, J.-F. Raymond. Linear colorings of graphs. Preprint, 2025. `https://arxiv.org/abs/2505.02768`.

[2] J. Kun, M. P. O'Brien, M. Pilipczuk, and B. D. Sullivan, Polynomial treedepth bounds in linear colorings, *Algorithmica* 83 (2021) 361–386.

Layered tree-independence number and clique-based separators

C. Dallard[(1)], M. Milanič[(2)], A. Munaro[(3)], S. Yang[(4)]

[(1)] University of Fribourg, Switzerland
[(2)] University of Primorska, Slovenia
[(3)] University of Parma, Italy
[(4)] Queen's University Belfast, UK

Motivated by a question of Galby, Munaro, and Yang (SoCG 2023) asking whether every graph class of bounded layered tree-independence number admits clique-based separators of sublinear weight, we investigate relations between layered tree-independence number, weight of clique-based separators, clique cover degeneracy and independence degeneracy. In particular, we provide a number of results bounding these parameters on geometric intersection graphs. For example, we show that the layered tree-independence number is $\mathcal{O}(g)$ for g-map graphs, $\mathcal{O}(\frac{r}{\tanh r})$ for hyperbolic uniform disk graphs with radius r, and $\mathcal{O}(1)$ for spherical uniform disk graphs with radius r. Our structural results have algorithmic consequences. In particular, we obtain a number of subexponential or quasi-polynomial-time algorithms for weighted problems such as Max Weight Independent Set and Min Weight Feedback Vertex Set on several geometric intersection graphs. Finally, we conjecture that every fractionally tree-independence-number-fragile graph class has bounded independence degeneracy.

Hitting all longest paths in hereditary graph classes

Amir Nikabadi[1]

[1] IT University of Copenhagen, Denmark.

The *longest path transversal number* of a connected graph G, is the minimum size of a set of vertices of G hitting all longest paths in G. Surprisingly, it is not known whether there exists a constant upper bound for the longest path transversal number of connected graphs. We present constant upper bounds for the longest path transversal number of multiple *hereditary classes of graphs*, that is, classes of graphs which are closed under induced subgraph containment, including the class of P_6-free graphs.
Based on a joint work with Paloma T. Lima and Paweł Rzążewski.

A Faster Algorithm for Independent Cut

V. Chernyshev[(1)], J. Rauch[(1)], D. Rautenbach[(1)], L. Redina[(2)]

[(1)] Ulm University, Ulm, Germany
[(2)] HSE University, Moscow, Russia

The previously fastest algorithm for deciding the existence of an independent cut had a runtime of $O^*(1.4423^n)$, where n is the order of the input graph. We improve this to $O^*(1.4143^n)$. In fact, we prove a runtime of $O^*\left(2^{(\frac{1}{2}-\alpha_\Delta)n}\right)$ on graphs of order n and maximum degree at most Δ, where $\alpha_\Delta = \frac{1}{2+4\lfloor\frac{\Delta}{2}\rfloor}$. Furthermore, we show that the problem is fixed-parameter tractable on graphs of order n and minimum degree at least βn for some $\beta > \frac{1}{2}$, where β is the parameter.

MIS on graphs excluding induced substructures

A. Wesolek(1)

(1) University of Potsdam, Potsdam, Germany

The complexity of the Maximum Independent Set problem is fully classified for graph classes defined by forbidden subgraphs or minors. It is polynomial-time solvable when excluding a forest in which every tree has at most three leaves as a subgraph, or when excluding a planar graph as a minor; in all other cases, it remains NP-hard. For graph classes defined by forbidden induced subgraphs or induced minors, however, the complexity landscape is largely unresolved. This talk presents some of the resolved cases ([1], [2]) and their connections to related problems.

References

[1] É. Bonnet, J. Duron, C. Geniet, S. Thomassé, A. Wesolek, Maximum Independent Set when excluding an induced minor: $K_1 + tK_2$ and $tC_3 \uplus C_4$, *ESA*, 2023 pp. 23:1–23:15.

[2] M. Bonamy, É. Bonnet, H. Déprés, L. Esperet, C. Geniet, C. Hilaire, S. Thomassé, A. Wesolek, Sparse graphs with bounded induced cycle packing number have logarithmic treewidth, *J. Comb. Theory Ser. B*, 2024 pp. 215-249.

Algorithmically on vertices that belong to all, some and no minimum dominating set in a tree

Radosław Ziemann[(1)], Paweł Żyliński[(1)]

[(1)] University of Gdańsk, 80-308 Gdańsk, Poland

A subset D of V_G is said to be a *dominating set* of a graph $G = (V_G, E_G)$ if each vertex in the set $V_G \setminus D$ has a neighbour in D. The (*independent*) *domination number* of G, denoted by $\gamma(G)$ (resp., by $\gamma_i(G)$), is defined to be the minimum cardinality of a (independent) dominating set D of G, and any minimum (independent) dominating set of G is referred to as a γ-set (resp., as a γ_i-set).

We propose a linear time algorithm for determining the sets of vertices that belong to all, some and no minimum dominating set of a tree, respectively, thus improving the quadratic time algorithm of Benecke and Mynhardt in 2008 [S. Benecke, C.M. Mynhardt, Trees with domination subdivision number one, *Australasian Journal of Combinatorics* 42, 201-209 (2008)].

Our result immediately implies the following corollaries: *For any tree T, the following problems are solvable in linear time and space: (A) The problem of verifying whether T is γ-excellent; (B) The problem of verifying whether* $\mathrm{sd}_\gamma(T) = 1$*; (C) The problem of verifying whether* $\mathrm{sd}_{\gamma_i}(T) = 1$. Recall that for a given graph parameter μ, the μ-subdivision number sd_μ is defined to be the minimum number of edges that must be subdivided to change μ, where each edge may be subdivided at most once.

Shiftable Heffter Spaces

M. Buratti[1], A. Pasotti[2]

[1] Sapienza Università di Roma, Roma, Italy
[2] Università degli Studi di Brescia, Brescia, Italy

The notion of a Heffter array [1] is equivalent to a pair of orthogonal Heffter systems. In [2, 3] we proved the existence of a set of r mutually orthogonal Heffter systems for any r. Such a set is equivalent to a resolvable partial linear space of degree r whose parallel classes are Heffter systems: we call such a design a *Heffter space*. In this talk we focus on *shiftable* Heffter spaces [4] presenting a direct construction, making use of pandiagonal magic squares, and a recursive one.

References

[1] D.S. Archdeacon, Heffter arrays and biembedding graphs on surfaces, *Electron. J. Combin.* 2015 #P1.74.

[2] M. Buratti, A. Pasotti, Heffter spaces, *Finite Fields Appl.* 2024 Article number 102464.

[3] M. Buratti, A. Pasotti, More Heffter spaces via finite fields, *J. Combin. Des.* 2025 pp.188-194.

[4] M. Buratti, A. Pasotti, Shiftable Heffter spaces, *Des. Codes Cryptog.* 2025 doi: 10.1007/s10623-025-01657-1.

On the Buratti-Horak-Rosa Conjecture for Small Supports

O. Ağırseven[1], M.A. Ollis[2]

[1] Independent Scholar, Somerville, MA, USA
[2] Emerson College, Boston, MA, USA

Label the vertices of the complete graph K_v with the distinct elements of $\mathbb{Z}_v$ and define the *length* ℓ of each edge as the cyclical distance between labels of its end-vertices. A Hamiltonian path through K_v is called a *realization* of a given multiset L if its edge labels are L. The *Buratti-Horak-Rosa Conjecture* is that there is a realization for a multiset L if and only if for any divisor d of v the number of multiples of d in L is at most $v - d$.

The toroidal lattice of vertices associated with each multiset was shown to be useful for constructing special types of realizations, the concatenations of which yield realizations for larger multisets [1, 2, 3, 4]. We will present our recent constructions yielding “standard linear realizations" for multisets with support of the form $\{1, x, y\}$ whenever the number of 1-edges is at least $\max(x, y) + \gcd(x, y) - 1$. These constructions considerably extend the parameters for which the conjecture is known to hold.

References

[1] O. Ağırseven, M. A. Ollis, Grid-based graphs, linear realizations and the Buratti-Horak-Rosa Conjecture, *submitted.*

[2] O. Ağırseven, M. A. Ollis, A Coprime Buratti-Horak-Rosa Conjecture and grid-based linear realizations, *submitted.*

[3] O. Ağırseven, M. A. Ollis, Construction techniques for linear realizations of multisets with small support, *accepted.*

[4] O. Ağırseven, M. A. Ollis, Standard and perfect linear realizations and the Buratti-Horak-Rosa Conjecture, *accepted.*

Gregarious hypergraph designs

P. Bonacini(1), L. Marino(1), Z. Tuza(2)

(1) University of Catania, Catania, Italy
(2) University of Pannonia, Veszprém, Hungary

A subhypergraph $\mathcal{H}$ of a given hypergraph is said to be gregarious with respect to a fixed vertex partition if the vertices of $\mathcal{H}$ belong to mutually distinct vertex classes. For graphs, this notion was introduced in the context of edge decomposition more than two decades ago, but its hypergraph generalization was first considered as late as in 2023, and only to the extent of just one theorem on 3-uniform hypergraphs. We begin the systematic study of gregarious decompositions of hypergraphs, with focus on complete n-partite r-uniform hypergraphs. Beyond their gregarious decompositions, a new approach is also offered and a related parameter introduced, expressing the gregarious decomposability of the blowups of a hypergraph.

References

[1] E. J. Billington, D. G. Hoffman, *Decomposition of complete tripartite graphs into gregarious 4-cycles*, Discrete Math. **261** (2003), 87–111.

[2] P. Bonacini, L. Marino, Z. Tuza, *Gregarious Hypergraph Designs*, work in progress.

[3] Zs. Tuza, *Gregarious decompositions of complete equipartite graphs and related structures*, Symmetry **15**:12 (2023), paper 2097, 14 pp.

Packing designs with large block size

Andrea Burgess[(1)], Peter Danziger[(2)], Daniel Horsley[(3)], Muhammad Tariq Javed[(2)]

[(1)] University of New Brunswick, Saint John, Canada
[(2)] Toronto Metropolitan University, Toronto, Canada
[(3)] Monash University, Melbourne, Australia

Given positive integers v, k, t and λ with $v \geq k \geq t$, a *packing design* $\mathrm{PD}_\lambda(v,k,t)$ is a pair $(V,\mathcal{B})$, where V is a v-set and $\mathcal{B}$ is a collection of k-subsets of V such that each t-subset of V appears in at most λ elements of $\mathcal{B}$. The maximum size of a $\mathrm{PD}_\lambda(v,k,t)$ is called the *packing number* and denoted $\mathrm{PDN}_\lambda(v,k,t)$.

We prove that for a positive integer n, $\mathrm{PDN}_\lambda(v,k,t) = n$ whenever $nk-(t-1)\binom{n}{\lambda+1} \leq \lambda v < (n+1)k-(t-1)\binom{n+1}{\lambda+1}$. For fixed t and λ, this determines the value of $\mathrm{PDN}_\lambda(v,k,t)$ when k is large with respect to v. By showing that if no point appears in more than three blocks, the blocks of a $\mathrm{PDN}_2(v,k,2)$ can be directed so that no ordered pair appears more than once, we also extend our results to directed packings with index $\lambda = 1$ and strength $t = 2$.

Hamiltonian Cycles on Coverings

Amanda Lynn Chafee[(1)], Brett Stevens[(1)]

[(1)] Carleton University, Ottawa, ON, Canada

A covering design is a v-set V and a list B of b blocks of size k where every pair from V must occur in at least one block. A 1-block intersection graph (1-BIG) is a graph $G = (B, E)$, where $b \in B$ and $(b, b') \in E$ if $|b \cap b'| = 1$ for $b, b' \in B$. This talk will go over what independence sets look like in a 1-BIG based on coverings with $k = 3$. We prove that optimal $k = 3$ coverings $v \equiv 5 \pmod 3$ have a Hamiltonian cycle and show why this proof fails for even v that are not Steiner Triple Systems.

References

[1] V. Chvátal, P. Erdös, A Note on Hamiltonian Circuits, *Discrete Math.* 1972 pp.111-113.

[2] P. Horák, A. Rosa, Decomposing Steiner Triple Systems into Small Configurations, *ARS Comb.* 1988 pp.91-105.

On the typical full automorphism group of Biembeddings of Archdeacon type

Simone Costa[1]

[1] University of Brescia, Brescia, Italy

In his seminal paper [1], Archdeacon introduced Heffter arrays as a tool to construct explicit $\mathbb{Z}_v$-regular biembeddings of complete graphs K_v into orientable surfaces. The automorphism groups of these embeddings were later investigated in [3], where upper bounds on their size were established, and in [2], where it was shown that these bounds are attainable.

In this talk, we consider a generalization of the notion of Heffter array: the *quasi*-Heffter arrays. This framework yields 2-colorable Archdeacon-type embeddings of the complete multipartite graph $K_{\frac{v}{t}\times t}$ into orientable surfaces. We then show that the automorphism group of such embeddings is, in the generic case, precisely $\mathbb{Z}_v$.

References

[1] D. Archdeacon, Heffter arrays and biembedding graphs on surfaces, *Electron. J. Combin.*, 22 (2015) #P1.74.

[2] S. Costa and L. Mella, A class of highly symmetric Archdeacon embeddings, *Australas. J. Combin.*, 88 (2024), 294–307.

[3] S. Costa and A. Pasotti, On the number of non-isomorphic (simple) k-gonal biembeddings of complete multipartite graphs, *Ars Math. Contemp.*, 24 (2024) #P4.02.

Switching operation for 2-designs and Hadamard matrices

D. Crnković[(1)], A. Švob[(1)]

[(1)] University of Rijeka, Rijeka, Croatia

In this talk, we will present the switching operation for 2-designs introduced in [1]. This switching operation can be adapted for switching Hadamard matrices. Further, we show that this switching can be applied to any Bush-type Hadamard matrix.

References

[1] D. Crnković, A. Švob, Switching for 2-designs, *Des. Codes Cryptogr.* 90 (2022), 1585–1593.

Graham's rearrangement for a class of semidirect products

Simone Costa(1), Stefano Della Fiore(1), Eva R. Engel(2)

(1) Università degli Studi di Brescia, Brescia 25123, Italy

(2) Princeton University, Princeton, NJ 08544, USA

A famous conjecture of Graham stated in 1971 asserts that for any set $A \subseteq \mathbb{Z}_p \setminus \{0\}$ there is an ordering $a_1, \ldots, a_{|A|}$ of the elements of A such that the partial sums $a_1, a_1+a_2, \ldots, a_1+a_2+\ldots+a_{|A|}$ are all distinct. Before the recently improvements, the state of the art was essentially that the conjecture holds when $|A| \leq 12$ and when A is a non-zero sum set of size $p-1$, $p-2$ or $p-3$. Many of the arguments for small A use the Polynomial Method and rely on Alon's Combinatorial Nullstellensatz. Very recently, Kravitz in [3], using a rectification argument, made a significant progress proving that the conjecture holds whenever $|A| \leq \log p/\log\log p$. A subsequent paper of Bedert and Kravitz [1] improved the logarithmic bound into a super-logarithmic one that is of the form $e^{c(\log p)^{1/4}}$ for some small constant $c > 0$.

In [2], we use a similar procedure to obtain an upper bound of the same type in the case of semidirect products $\mathbb{Z}_p \rtimes_\varphi H$ where $\varphi : H \to Aut(\mathbb{Z}_p)$ satisfies $\varphi(h) \in \{id, -id\}$ for each $h \in H$ and where H is abelian and each subset of H can be ordered such that all of its partial products are distinct.

References

[1] B. Bedert, N. Kravitz. Graham's rearrangement conjecture beyond the rectification barrier, *Israel J. Math.*, (2025).

[2] S. Costa, S. Della Fiore and E. R. Engel. Graham's rearrangement for a class of semidirect products, *arXiv*, (2025).

[3] N. Kravitz. Rearranging small sets for distinct partial sums, *Integers*, 24 (2024).

Embedding partial Latin squares in Latin squares with many mutually orthogonal mates

Diane Donovan[(1)], Mike Grannell, E. Şule Yazıcı

[(1)] SMP, The University of Queensland, Brisbane, Australia

In this talk, I will review combinatorial constructions developed by Donovan, Grannell, and Yazıcı that verify that a pair of (partial) orthogonal Latin squares of order n, can be embedded in a set of $t+2$ mutually orthogonal Latin squares (MOLS) of polynomial order in n, for any $t \geq 2$. Notably, this construction verifies, for the first time, the existence of a set of nine MOLS of order 576, improving upon the earlier maximum of eight.

If time permits, I will also present earlier work by Donovan and Yazıcı, which provides the first constructive, polynomial-order embedding for a pair of orthogonal partial Latin squares.

G-designs for some graphs on seven edges

D. Banegas[(1)], A. Carlson[(1)], D. Froncek[(1)]

[(1)] University of Minnesota Duluth, Duluth, USA

A *G-design of order n* is a collections of s edge disjoint graphs G_i isomorphic to G, whose union forms the complete graph K_n.

We complement a recent result by Fronček and Kubesa [2] by examining the remaining three disconnected bipartite graphs with seven edges: on nine and ten vertices. While the result itself is not too exciting, it provides an opportunity to present several different methods for finding G-designs. For $n \equiv 0, 1 \pmod{14}$, we use classical labeling methods introduced by Rosa [4] and generalized by El-Zanati et al. [3] and Bunge [1].

For $n \equiv 7 \pmod{14}$, we first decompose K_{14k+7} into $K_{7,7}$ and $K_{14} - K_7$ and then each of them to G. For $n \equiv 8 \pmod{14}$, we decompose K_{14k+8} into the circulant $Cir(14k+8; 1, 2, 7k+3, 7k+4)$ and its complement and then use labelings for the complement and labelings with some adjustments for the circulant.

References

[1] R. Bunge, On 1-rotational decompositions of complete graphs into tripartite graphs, *Opuscula Math.* **39**(5) (2019), 624–643.

[2] D. Froncek and M. Kubesa, Decomposition… *Commun. Comb. Optim.*, `https://doi.org/10.22049/cco.2024.29264.1915`.

[3] S. El-Zanati, C. Vanden Eynden, and N. Punnim, On the cyclic decomposition of complete graphs into bipartite graphs, *Australas. J. Combin.* **24** (2001), 209–219.

[4] A. Rosa, *On certain valuations of the vertices of a graph*, In: Theory of Graphs (Intl. Symp. Rome 1966), Gordon and Breach, Dunod, Paris, 1967, 349–355.

Steiner triple systems with Veblen points

Mario Galici(1)

(1) University of Salento, Lecce, Italy

A Steiner triple system, STS(v), is a $2-(v,3,1)$ design. A *Veblen point* of an STS is a point for which any two other distinct points generate a Pasch configuration. Steiner triple systems given by the point-line design of a projective space PG($n, 2$) are precisely those in which every point is a Veblen point.

Steiner loops provide a natural algebraic framework for studying Steiner triple systems. We focus on their *Schreier extensions*, which offer an effective method for constructing Steiner triple systems with Veblen points. This concept was first introduced for loops in general in [1], and later explored in the context of Steiner loops in [2]. In particular, in [3] we investigate Veblen points in Steiner triple systems of orders 19, 27, and 31, determining their number and giving concrete examples.

References

[1] P. T. Nagy, K. Strambach, Schreier Loops, *Czechoslovak Mathematical Journal.* 2008.

[2] G. Falcone, A. Figula, M. Galici, Extensions of Steiner Triple Systems, *Journal of Combinatorial Designs.* 2025.

[3] G. Filippone, M. Galici, On the number of small Steiner triple systems with Veblen points, *Discrete Mathematics.* 2025.

On decomposition thresholds for odd-length cycles

Darryn Bryant[(1)], Peter Dukes[(2)], Daniel Horsley[(3)], Barbara Maenhaut[(1)], Richard Montgomery[(4)]

[(1)] University of Queensland, Brisbane, Australia
[(2)] University of Victoria, Victoria, Canada
[(3)] Monash University, Melbourne, Australia
[(4)] University of Warwick, Warwick, UK

An (edge) *decomposition* of a graph G is a set of subgraphs of G whose edge sets partition the edge set of G. I will discuss our recent proof that, for each odd $\ell \geq 5$, any graph G of sufficiently large order n with minimum degree at least $\big(\frac{1}{2} + \frac{1}{2\ell-4} + o(1)\big)n$ has a decomposition into ℓ-cycles if and only if ℓ divides $|E(G)|$ and each vertex of G has even degree. This threshold cannot be improved beyond $\frac{1}{2} + \frac{1}{2\ell-2}$. It was previously shown that the thresholds approach $\frac{1}{2}$ as ℓ becomes large, but our thresholds do so significantly more rapidly. Our methods can be applied to tripartite graphs more generally and we also obtain some bounds for decomposition thresholds of other tripartite graphs.

On relative simple Heffter spaces

L. Johnson[(1)], L. Mella[(2)], A. Pasotti[(3)]

[(1)] University of Bristol, Bristol, United Kingdom
[(2)] University of Modena and Reggio Emilia, Modena, Italy
[(3)] University of Brescia, Brescia, Italy

Let G be an abelian group and suppose that J is a subgroup of G of order t, a *half-set* V of $\mathrm{G}\backslash J$ is a subset of $\mathrm{G}\backslash J$ such that for each non-involution element $x \in \mathrm{G}\backslash J$, either x or $-x$ is contained in V and any involution elements of $\mathrm{G}\backslash J$ are also contained in V. An $(nk, k)_t$ *relative Heffter system* is a partition of a half-set V of $\mathrm{G}\backslash J$ into zero-sum blocks of equal size. Two $(nk, k)_t$ relative Heffter systems $\mathcal{P}$ and $\mathcal{Q}$, based on the same half-set, are said to be *orthogonal* if their blocks intersect in at most one element. In a [1], we introduce the concept of a $(nk, k; r)_t$ *relative Heffter space*, which is a collection of r mutually orthogonal Heffter systems. This definition naturally generalises the concepts of a relative Heffter array and a Heffter space.

The *density* of a Heffter space refers to the density of the collinear graph associated with the Heffter space. In the paper this talk is based on [1], we present two constructions of infinite families of relative Heffter spaces, that satisfy the additional property of being globally simple. One of these constructions always achieves maximal density. By obtaining results on globally simple Heffter spaces, [1] also obtains new results on mutually orthogonal cycle decompositions and biembeddings of cyclic cycle decompositions of the complete multipartite graph into an orientable surface.

References

[1] L. Johnson, L. Mella, A. Pasotti, On relative simple Heffter spaces, arXiv preprint arXiv:2503.07445, (2025).

On directed Oberwolfach problem with tables of even lengths

Alice Lacaze-Masmonteil[1]

[1] Department of Mathematics and Statistics, University of Regina

A $(\vec{C}_{m_1}, \vec{C}_{m_2}, \ldots, \vec{C}_{m_t})$-factor of a directed graph G is a spanning subdigraph of G comprised of t disjoint directed cycles of lengths $m_1, m_2, \ldots, m_t$, where $m_i \geqslant 2$. In this talk, we will be constructing a decomposition of the complete symmetric digraph K_{2n}^* into $(\vec{C}_{m_1}, \vec{C}_{m_2}, \ldots, \vec{C}_{m_t})$-factors when $m_1 + m_2 + \cdots + m_t = 2n$, $t \geqslant 3$, and n is odd. The existence of this decomposition implies a complete solution to the directed Oberwolfach problem with t tables of even lengths and $2n$ guests such that n is odd. This is joint work with Andrea Burgess and Peter Danziger.

Further constructions of square integer relative Heffter arrays

Diane Donovan[(1)], Sarah Lawson[(1)], James Lefevre[(1)]

[(1)] The University of Queensland, Brisbane, Australia

Heffter arrays are a fascinating combinatorial object introduced by Archdeacon in 2015 [1] and later generalized to *relative* Heffter arrays by Costa et al. in 2019 [2]. In this talk, I will focus on *square integer relative Heffter arrays*, which are $n \times n$ arrays where each row and column contains the same number of entries, the sum of each row and column is zero and where, given the subgroup J of size t, every nonzero element of $\mathbb{Z}_{2nk+t} \setminus J$ appears exactly once up to sign. There are many open problems regarding the existence of these arrays. I will focus on arrays that contain *a primary transversal*, a transversal of the set $\{1, \ldots, n\}$ up to sign. I will present a new family of square integer relative Heffter arrays along with complete results for their existence for n prime and $k = 3$[3].

References

[1] D.Archdeacon, Heffter arrays and biembedding graphs on surfaces, *The Electronic Journal of Combinatorics, 22(1)* 2015 P1.74.

[2] S.Costa, F.Morini, A.Pasotti, M.Pellegrini, A generalization of Heffter arrays *Journal of Combinatorial Designs, 28(3)* 2019 P171-206.

[3] D.Donovan, S.Lawson, J.Lefevre, Further constructions of square integer relative Heffter arrays, *in preparation.*

Balanced generalized kite designs

Paola Bonacini(1), Lucia Marino(1)

(1) University of Catania — Department of Mathematics and Informatics, Italy

In this work, we study valuations and labelings of bipartite graphs and their applications to cyclic graph designs. In particular, we introduce the notion of (A, B)-ordered and uniformly ordered labelings for a bipartite graph $G = (V, E)$ with partition classes A and B. Using these labelings, we construct (A, B)-uniformly ordered labelings and describe how shifts of the labeling modulo $r + 1$ preserve certain ordering properties.

Our main result is a constructive method for cyclic $(C_m + P_{n+1})$-designs of order v, where $v \equiv 1 \mod 2(m + n)$. These results illustrate the interplay between ordered labelings of bipartite graphs and the construction of balanced generalized kite designs.

References

[1] L. Berardi, M. Gionfriddo, R. Rota, Balanced and strongly balanced P_k-designs, *Discrete Math.* 312: 633–636, 2012.

[2] J. C. Bermond, C. Huang, A. Rosa, D. Sotteau, Decompositions of complete graphs into isomorphic subgraphs with five vertices, *Ars Combin.* 10, 211–254, 1980.

[3] P. Bonacini, M. Gionfriddo, L. Marino, Balanced House-systems and Nestings, *Ars Combin.* 121, 429–436, 2015.

[4] P. Bonacini, M. Gionfriddo, L. Marino, Locally Balanced G-designs, *Mathematics* 2, 408, 2023.

[5] P. Bonacini, M. Gionfriddo, L. Marino, A survey on recent results on Balanced Graph and Hypergraph Designs, *Le Matematiche* LXXIX, 449-467, 2024.

[6] P. Bonacini, M. Di Giovanni, M. Gionfriddo, L. Marino, A. Tripodi, The spectrum of balanced $P^{(3)}(1,5)$-designs, *Contrib. Discrete Math.* 12(1), 133–142, 2017.

[7] A. Bonisoli, B. Ruini, Tree-designs with balanced-type conditions, *Discrete Math.* 313(11), 1197–1205, 2013.

[8] S. Bonvicini, Degree and orbit balanced Γ-designs when Γ has five vertices, *J. Combin. Des.* 21(9), 359–389, 2013.

[9] M. Gionfriddo, S. Küçükçifçi, L. Milazzo, Balanced and strongly balanced 4-kite designs, *Util. Math.* 91, 121–129, 2013.

Cycle decompositions of circulants $C(n, \{1, 3\})$

Juliana Palmen[(1)]

[(1)] AGH University, Cracow, Poland

A *decomposition* of a graph G is a collection of edge-disjoint subgraphs $H_1, H_2, \ldots, H_t$ of G such that each edge of G belongs to exactly one H_i. We call this collection a *k-factorization* when every H_i is a k-regular spanning subgraph of G.

For a positive integer n and a set $S \subseteq \{1, \ldots, \lfloor(\frac{n}{2}\rfloor)\}$ a *circulant* $C(n, S)$ is a graph $G = (V, E)$ such that $V = \mathbb{Z}_n$ and $E = \{\{u, v\} : \delta(u, v) \in S\}$ where $\delta(u, v) = min\{\pm|u - v| \pmod n)\}$.

Some results on decomposition of those graphs into cycles were obtained. Inspired by the work of Bryant and Martin [1], who gave a complete solution for the cycle decomposition of $C(n, \{1, 2\})$, we examine the case when $S = \{1, 3\}$. Among others, we present the results on decomposition of $C(n, \{1, 3\})$ into cycles of odd lengths and into cycles of even lengths.

In [2] Bryant showed that, whenever $n \geq 5$, there exists a 2-factorization of $C(n, \{1, 2\})$ in which one factor is a Hamiltonian cycle and the other factor is isomorphic to any given 2-regular graph of order n. We discuss some open problems concerning the 2-factorization of $C(n, \{1, 3\})$.

References

[1] E.D.Bryant, M.Geoffrey, Some results on decompositions of low degree circulant graphs, *Australas. J Comb. 45* 2009 pp.251-262.

[2] D.Bryant, Hamilton cycle rich two-factorizations of complete graphs, *Journal of Combinatorial Designs 12.2* 2004 pp.147-155.

Constructing magic objects

M.A. Pellegrini[(1)]

[(1)] Università Cattolica del Sacro Cuore, Brescia, Italy

In this talk I will describe some techniques that can be used to construct different magic objects such as signed magic arrays $\mathrm{SMA}(m,n;s,k)$, magic rectangle sets $\mathrm{MRS}(m,n;s,k;c)$ and integer Heffter arrays $\mathrm{H}(m,n;s,k)$. In the first two cases, we have determined necessary and sufficient conditions for their existence (see [1, 2]), while for integer Heffter arrays the problem is still open for some small values of k (see [4]). Also, I will describe some constructions of Γ-magic rectangle sets $\mathrm{MRS}_\Gamma(m,n;s,k;c)$, where Γ is a finite abelian group (see [3]).

References

[1] F. Morini, M.A. Pellegrini, Magic partially filled arrays on abelian groups, *J. Combin. Des.* 2023(31) pp. 347-367.

[2] F. Morini, M.A. Pellegrini, Signed magic arrays: existence and constructions, *Preprint.*

[3] F. Morini, M.A. Pellegrini, S. Sora, On a conjecture by Sylwia Cichacz and Tomasz Hinc, and a related problem, *Discrete Appl. Math.* 2025(367) pp. 53-67.

[4] M.A. Pellegrini, T. Traetta, Towards a solution of Archdeacon's conjecture on integer Heffter arrays, *J. Combin. Des.* 2025(33) pp. 310-323.

On large Sidon sets

Ingo Czerwinski[(1)], Alexander Pott[(1)]

[(1)] Otto von Guericke University Magdeburg, Germany

A (binary) Sidon set M is a subset of $\mathbb{F}_2^t$ such that the sum of four distinct elements of M is never 0. The goal is to find Sidon sets of large size. In this talk we show that the graphs of almost perfect nonlinear (APN) functions with high linearity can be used to construct large Sidon sets. Thanks to recently constructed APN functions [[1, 2]]$\mathbb{F}_2^8 \to \mathbb{F}_2^8$ with high linearity, we can construct Sidon sets of size 192 in $\mathbb{F}_2^{15}$, where the largest sets so far had size 152. This result will be published in the Journal of Combinatorial Theory (A).

References

[1] Beierle, Christof and Leander, Gregor and Perrin, Léo, Trims and extensions of quadratic APN functions, *Designs, Codes and Cryptography* 2022 pp.1009-1036.

[2] Beierle, Christof and Leander, Gregor, New Instances of Quadratic APN Functions, *IEEE Transactions on Information Theory* 2020 pp. 670-678.

Exploring the Oberwolfach problem through solutions with non trivial automorphism group

Gloria Rinaldi[(1)]

[(1)] University of Modena and Reggio Emilia, Italy

The Oberwolfach Problem, originally posed by Ringel in 1967, asks for a decomposition of complete graphs into 2-factors of prescribed isomorphism type. A recent asymptotic result (2021, Glock et al.) guarantees the existence of solutions provided that the total number of vertices is sufficiently large, yet, the possibility of determining explicit and constructible solutions for every configuration remains an open problem. Searching for solutions with a non trivial automorphism group may aid in constructing explicit examples, both in the classical setting and in variants obtained by adding or removing a repeatd $1-$factor from the complete graph.

Distinct Difference Configurations

Emma Smith[(1)]

[(1)] Royal Holloway, University of London, United Kingdom

A subset D of a group is a Distinct Difference Configuration (DDC) if the differences $g^{-1}h$ are distinct, where g and h range over all (ordered) pairs of distinct elements of D. When developed over the group, the resulting blocks form a $2-(|G|,|D|,1)$ packing.

DDCs also have practical applications in key predistribution schemes for wireless sensor networks, especially when the network's communication structure mirrors the Cayley graph of a group. In such a scheme, using a bounded DDC ensures an efficient trade-off between local connectivity and global security.

This talk will also cover joint work with Luke Stewart and Simon Blackburn where we show that large, bounded DDCs exist in free groups, extending the theory beyond abelian or finite settings.

References

[1] S. Blackburn, E. Smith, L. Stewart, Subsets of free groups with distinct differences, *Comb. Theory.* Accepted. Available at arXiv:2307.03627.

On some new regular digraphs from finite groups

A. Švob(1), A. E. Brouwer, D. Crnković(1), T. Zrinski(1), M. Zubović Žutolija(1)

(1) University of Rijeka, Rijeka, Croatia

In this talk, we describe a construction of certain regular digraphs using finite simple groups. We introduce the notion of orbit matrices of digraphs and point out some interesting results obtained using specific linear groups. In particular, we present the first example of a directed strongly regular graph with parameters (63,11,8,1,2) along with several other new directed strongly regular graphs obtained from finite simple groups. The talk is based on the papers [1, 2, 3].

References

[1] A. E. Brouwer, D. Crnković, A. Švob, A construction of directed strongly regular graphs with parameters (63,11,8,1,2), *Discrete Math.* 347 (2024), 114146, 3 pages.

[2] A. E. Brouwer, D. Crnković, A. Švob, M. Zubović Žutolija, Some directed strongly regular graphs constructed from linear groups, *Appl. Algebra Engrg. Comm. Comput.*, to appear.

[3] D. Crnković, A. Švob, T. Zrinski, Construction of directed strongly regular graphs via their orbit matrices and genetic algorithm, `https://arxiv.org/abs/2412.14787`.

An explicit lower bound on the largest cycle for the solvability of the Oberwolfach problem

T. Traetta[(1)]

[(1)] University of Brescia, Italy

The Oberwolfach problem OP(F), posed by Ringel in 1967, asks for a decomposition of the complete graph K_v into copies of a given 2-regular graph F of odd order v. Some recent non-constructive results [2, 4] provide an asymptotic proof of the solvability of OP(F) for sufficiently large orders, but leave the specific lower bound for v unquantified.

In this talk, we present a method to build solutions to OP(F) whenever F has a cycle of length greater than an explicit lower bound [5], thereby partially filling this gap. Our constructions combine the amalgamation-detachment technique [3] with methods for building suitable decompositions of K_v having an automorphism group with a nearly-regular action on the vertices [1].

References

[1] A.C. Burgess, P. Danziger, T. Traetta, On the Oberwolfach problem for single-flip 2-factors via graceful labelings, *J. Combin. Theory Ser. A* 189(2022) pp.105-611.

[2] S. Glock, F. Joos, J. Kim, D. Kühn, D. Osthus., Resolution of the Oberwolfach problem, *J. Eur. Math. Soc.* 23(2021) pp.2511-2547.

[3] A.J.W. Hilton, M. Johnson, Some results on the Oberwolfach problem, *J. London Math. Soc.* 64(2001) pp.513-522.

[4] P. Keevash, K. Staden, The generalised Oberwolfach problem, *J. Combin. Theory B* 152(2022) pp.281–318.

[5] T. Traetta, A constructive solution to the Oberwolfach problem with a large cycle, *Discrete Math.* 347(2024) 113947.

Domination type parameters in 3-regular and 4-regular graphs

Michael A. Henning(1)

(1) University of Johannesburg, Johannesburg, South Africa

A set S of vertices in a graph G is a dominating set if every vertex in $V(G) \setminus S$ is adjacent to a vertex in S. The domination number, $\gamma(G)$, of G is the minimum cardinality among all dominating set in G. We discuss best possible upper bounds on domination-type parameters in cubic graphs. Among other results, we show that if G is a cubic graph of order n, then $\gamma_{\rm t2}(G) \le \frac{2}{5}n$ and $\gamma_r(G) \le \frac{2}{5}n$, where $\gamma_{\rm t2}(G)$ and $\gamma_r(G)$ denote the semitotal and restrained domination numbers, respectively. The $\frac{1}{3}$-conjecture for domination in 4-regular graphs states that if G is a 4-regular graph of order n, then $\gamma(G) \le \frac{1}{3}n$. We prove this conjecture when G has no induced 4-cycle. A thorough treatise on dominating sets can be found in [1, 2, 3].

References

[1] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning (eds), *Topics in Domination in Graphs.* Series: Developments in Mathematics, Vol. 64, Springer, Cham, 2020. viii + 545 pp.

[2] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning (eds), *Structures of Domination in Graphs.* Series: Developments in Mathematics, Vol. 66, Springer, Cham, 2021. viii + 536 pp.

[3] T. W. Haynes, S. T. Hedetniemi, and M. A. Henning, *Domination in Graphs: Core Concepts* Series: Springer Monographs in Mathematics, Springer, Cham, 2023. xx + 644 pp.

On proper secondary and multiple dominating sets

Paweł Bednarz[(1)], Adrian Michalski[(1)]

[(1)] Rzeszow University of Technology, Rzeszów, Poland

Let $k \geq 1$ be an integer. A subset $D \subset V(G)$ is (1,k)-dominating if for every vertex $v \in V(G) \setminus D$ there exist vertices $u, w \in D$ such that $uv \in E(G)$ and $d_G(v, w) \leq k$. If $k = 1$, then we obtain the definition of (1,1)-dominating sets, which are also known as 2-dominating sets. If $k = 2$, then we have the concept of (1,2)-dominating sets.

In [1] Michalski et al. introduced the concept of proper (1,2)-dominating sets to distinguish (1,2)-dominating sets from (1,1)-dominating sets. Formally, a *proper* (1,2)*-dominating set* is a (1,2)-dominating set that is not (1,1)-dominating. Basing on this idea, we considered proper (1,3)-dominating sets. Moreover, in [2, 3] proper l-dominating sets i.e. l-dominating sets which are not $(l+1)$-dominating were defined and studied.

In this talk we present some results concerning proper secondary and multiple dominating sets, in particular we focus on the problem of their existence. Moreover, we show relations between parameters of these types of domination.

References

[1] A. Michalski, I. Włoch, M. Dettlaff, M. Lemańska, *On proper (1,2)-dominating sets*, Mathematical Methods in the Applied Sciences, 45(11), (2022), 7050–7057.

[2] P. Bednarz, M. Pirga, *On proper 2-dominating sets in graphs*, Symmetry, 16, (2024), 296.

[3] D. Chen, S. Cai, *Proper 3-Dominating Sets in Graphs*, Mathematics, 13, (2025), 1960.

Isolation of graphs

Peter Borg[(1)]

[(1)] University of Malta, Msida, Malta

Given a set $\mathcal{F}$ of graphs, we call a copy of a graph in $\mathcal{F}$ an *$\mathcal{F}$-graph*. The *$\mathcal{F}$-isolation number* of a graph G, denoted by $\iota(G, \mathcal{F})$, is the size of a smallest subset D of the vertex set of G such that the closed neighbourhood $N[D]$ of D intersects the vertex sets of the $\mathcal{F}$-graphs contained by G (equivalently, $G - N[D]$ contains no $\mathcal{F}$-graph). When $\mathcal{F}$ consists of a 1-clique, $\iota(G, \mathcal{F})$ is the *domination number* of G. When $\mathcal{F}$ consists of a 2-clique, $\iota(G, \mathcal{F})$ is the *vertex-edge domination number* of G. The study of the general $\mathcal{F}$-isolation problem was introduced by Caro and Hansberg [4] in 2017. This study is expanding very rapidly. A brief account of its development and of the speaker's recent work in this field [1, 2, 3] will be provided.

References

[1] P.Borg, Isolation of regular graphs, stars and k-chromatic graphs, arXiv:2303.13709 [math.CO].

[2] P. Borg, Isolation of regular graphs and k-chromatic graphs, *Mediterr. J. Math.* 21 (2024) paper 148.

[3] P. Borg, Proof of a conjecture on isolation of graphs dominated by a vertex, *Discrete Appl. Math.* 371 (2025) pp.247-253.

[4] Y.Caro, A.Hansberg, Partial domination - the isolation number of a graph, *Filomat* 31 (2017) pp.3925-3944.

2-Rainbow Independent Domination in Complementary Prisms

Dragana Božović[(1)], Gordana Radić[(1,2)], Aleksandra Tepeh[(1,2)]

[(1)] University of Maribor, Faculty of Electrical Engineering and Computer Science, Koroška cesta 46, 2000 Maribor, Slovenia
[(2)] Institute of Mathematics, Physics and Mechanics, Jadranska ulica 19, 1000 Ljubljana, Slovenia

A function f that assigns values from the set $\{0, 1, 2\}$ to each vertex of a graph G is called a 2-rainbow independent dominating function, if the vertices assigned the value 1 form an independent set, the vertices assigned the value 2 form another independent set, and every vertex to which 0 is assigned has at least one neighbor in each of the mentioned independent sets. The weight of this function is the total number of vertices assigned nonzero values. The 2-rainbow independent domination number of G, $\gamma_{\mathrm{ri2}}(G)$, is the minimum weight of such a function.

We study the 2-rainbow independent domination number of the complementary prism $G\overline{G}$ of a graph G, which is constructed by taking G and its complement $\overline{G}$, and then adding edges between corresponding vertices. We provide tight bounds for $\gamma_{\mathrm{ri2}}(G\overline{G})$, and characterize graphs for which the lower bound, i.e. $\max\{\gamma_{\mathrm{ri2}}(G), \gamma_{\mathrm{ri2}}(\overline{G})\} + 1$, is attained.

References

[1] S. Brezovnik, T. Kraner Šumenjak, Complexity of k -rainbow independent domination and some results on the lexicographic product of graphs, *Appl. Math. Comp.* **349** (2019) 214–220.

[2] B. Gabrovšek, A. Peperko, J. Žerovnik, On the 2-rainbow independent domination numbers of some graphs, *Cent. Eur. J. Oper. Res.* **31(1)** (2023) 1–15.

[3] R. Hammack, W. Imrich, and S. Klavžar, *Handbook of Product Graphs: Second Edition*, CRC Press, Boca Raton, FL, 2011.

[4] T.W. Haynes, M.A. Henning, P.J. Slater and L.C. Van Der Merwe, The complementary product of two graphs, Bull. Instit. Combin. Appl. **51** (2007), 21–30.

[5] T. Kraner Šumenjak, D. F. Rall, A. Tepeh, On k-rainbow independent domination in graphs, *Appl. Math. Comp.* **333** (2018) 353–361.

[6] X. Shi, P. Wu, Z. Shao, V. Samodivkin, S.M. Sheikholeslami, M. Soroudi, and S. Wang, Changing and unchanging 2-rainbow independent domination, *IEEE Access* **7** (2019) 72604–72612.

[7] E. Zhu, An improved Nordhaus–Gaddum-type theorem for 2-rainbow independent domination number, *Mathematics* **9** (2021) 402.

An algorithmic proof for the domination number of grid graphs

Yao-Ting Shen(1), Chiuyuan Chen(1)

(1) National Yang Ming Chiao Tung University, Hsinchu, Taiwan

The *domination number* $\gamma(G)$ of a graph G is the minimum cardinality of a dominating set of G. Let the notion $[i]$ denote the set $\{1, 2, \ldots, i\}$. Let $G_{m,n}$ denote the complete (m, n) grid; i.e., the vertex set of $G_{m,n}$ is $[m] \times [n]$, and two vertices (i, j) and (i', j') are adjacent if $|i - i'| + |j - j'| = 1$. Reference [1] proves that for every $16 \leq m \leq n$, $\gamma(G_{m,n}) = \left\lfloor \frac{(m+2)(n+2)}{5} \right\rfloor - 4$ and thus concludes the calculation of $\gamma(G_{m,n})$ of all (m, n) grid graphs. In this study (a preliminary version was in [2]), we consider the charging pad deployment problem (CPDP) for wireless rechargeable sensor networks with grid topology $\mathcal{G}_{m,n}$. CPDP aims to find a deployment of charging pads for the unmanned aerial vehicle (UAV) so that every sensor node is covered by at least one pad and the number of pads is as small as possible. Note that $\mathcal{G}_{m,n} \cong G_{m+1,n+1}$. We show that when the grid length $\mathcal{L}$ satisfies $\frac{1}{\sqrt{2}} d_\theta < \mathcal{L} \leq \frac{2}{\sqrt{5}} d_\theta$, CPDP is related to finding $\gamma(\mathcal{G}_{m,n})$, where d_θ is the maximum flying distance of the UAV when its energy is θ (a pre-defined energy threshold). We propose a charging pad deployment scheme for $\mathcal{G}_{m,n}$ and prove that for every $15 \leq m \leq n$, our scheme uses the least number of pads; this provides an algorithmic proof for the domination number of grid graphs.

References

[1] D. Gonçalves, A. Pinlou, M. Rao, and S. Thomassé, The domination number of grids, *SIAM J. Discrete Math.*, vol. 25, no. 3, pp. 1443–1453, 2011.

[2] Y.-T. Shen, *Three charging pad deployment schemes for wireless rechargeable sensor networks with grid topology*, (Master thesis), National Yang Ming Chiao Tung University, Hsinchu, Taiwan, 2023.

2-Domination edge subdivision in trees

M. Dettlaff[(1)], M.A. Henning[(2)], M. Lemańska[(3)], A. Roux[(4)], J. Topp[(5)]

(1) University of Gdańsk, Gdańsk, Poland
(2) University of Johannesburg, Johannesburg, South Africa
(3) Gdańsk University of Technology, Gdańsk, Poland
(4) Stellenbosch University, Stellenbosch, South Africa
(5) University of Applied Sciences, Elbląg, Poland

A set S of vertices in a graph G is a 2-dominating set of G if every vertex not in S has at least two neighbors in S, where two vertices are neighbors if they are adjacent. The 2-domination number of G, denoted by $\gamma_2(G)$, is the minimum cardinality among all 2-dominating sets in G. A γ_2-set of G is a 2-dominating set of G of cardinality $\gamma_2(G)$. The 2-domination subdivision number of G, denoted by $\mathrm{sd}_2(G)$, is the minimum number of edges which must be subdivided in order to increase the 2-domination number. If T is a tree of order $n \geq 3$, then $\mathrm{sd}_2(T) \leq 2$. In [1] we show that $\mathrm{sd}_2(T) = 1$ if and only if the set of vertices that belong to no γ_2-set of G is nonempty. A graph G is γ_2-q-critical if q is the least number such that for every subset of edges S of cardinality q, the graph produced by subdivision of S has a greater 2-domination number. If T is a γ_2-q-critical tree of order $n \geq 3$, then we prove that $q \leq n-2$. Among other results, we characterize γ_2-q-critical trees when q is large, namely $q \in \{n-4, n-3, n-2\}$. We also characterize γ_2-1-critical trees [1] and γ_2-2-critical trees [2].

References

[1] M.Dettlaff, M.A.Henning, M.Lemańska, A.Roux, J.Topp, 2-Domination edge subdivision in trees, *Australas. J. Combin.* to appear.

[2] M.Dettlaff, M.A.Henning, M.Lemańska, A.Roux, 2-Domination critical trees upon edge subdivision, *Australas. J. Combin.* 92(3) (2025), 357—381.

Complexity of Defensive Domination

Steven Chaplick[(1)], Grzegorz Gutowski[(2)], Tomasz Krawczyk[(3)]

[(1)] Maastricht University
[(2)] Jagiellonian University
[(3)] Warsaw University of Technology

In a graph G, a *k-attack* A is any set of at most k vertices and *ℓ-defense* D is a set of at most ℓ vertices. We say that defense D *counters* attack A if each $a \in A$ can be matched to a distinct *defender* $d \in D$ with a equal to d or a adjacent to d in G. In the *defensive domination problem*, we are interested in deciding, for a graph G and positive integers k and ℓ given on input, if there exists an ℓ-defense that counters every possible k-attack on G. Defensive domination is a natural resource allocation problem and can be used to model network robustness and security, disaster response strategies, and redundancy designs.

The defensive domination problem is naturally in the complexity class Σ_2^{P}. The problem was known to be NP-hard in general, and polynomial-time algorithms were found for some restricted graph classes. In this note, we prove that the defensive domination problem is Σ_2^{P}-complete.

We also introduce a natural variant of the defensive domination problem in which the defense is allowed to be a multiset of vertices. This variant is also Σ_2^{P}-complete, but we show that it admits a polynomial-time algorithm in the class of interval graphs. A similar result was known for the original setting in the class of proper interval graphs.

References

[1] Steven Chaplick, Grzegorz Gutowski, and Tomasz Krawczyk. A Note on the Complexity of Defensive Domination, 2025. `https://arxiv.org/abs/2504.14390`.

On rainbow domination regular graphs

Boštjan Kuzman(1)

(1) University of Ljubljana, Slovenia

In [1], a d-regular graph X is called d-rainbow domination regular or d-RDR, if its d-rainbow domination number $\gamma_{rd}(X)$ attains the lower bound $n/2$ for d-regular graphs, where n is the number of vertices. In [2], some combinatorial constructions to construct new d-RDR graphs from existing ones are given and two general criteria for a vertex-transitive d-regular graph to be d-RDR are proven. A list of vertex-transitive 3-RDR graphs of small orders is then produced and their partial classification into families of generalized Petersen graphs, honeycomb-toroidal graphs and a specific family of Cayley graphs is given by investigating the girth and local cycle structure of these graphs. In the talk, some more recent results and open problems on d-RDR graphs will be presented.

References

[1] KUZMAN, Boštjan. On k-rainbow domination in regular graphs. *Discrete applied mathematics.* 2020, vol. 284, pp. 454 – 464.

[2] KUZMAN, Boštjan. On cubic rainbow domination regular graphs. *Discrete applied mathematics.* 2025, vol. 373, pp. 26 – 38.

Paired versus double domination in forbidden graph classes

Michael A. Henning[1], Paras Vinubhai Maniya[2],
Dinabandhu Pradhan[2]

[1] University of Johannesburg, Auckland Park, South Africa
[2] Indian Institute of Technology (ISM), Dhanbad, India

A set D of vertices in a graph G is a dominating set of G if every vertex not in D has a neighbor in D, where two vertices are neighbors if they are adjacent. If the dominating set D of G has the additional property that the subgraph induced by D contains a perfect matching (not necessarily as an induced subgraph), then D is a paired dominating set of G. The paired domination number of G, denoted by $\gamma_{pr}(G)$, is the minimum cardinality of a paired dominating set of G. A set $D \subseteq V(G)$ is a double dominating set of G if every vertex in $V(G) \setminus D$ has at least two neighbors in D, and every vertex in D has a neighbor in D. The double domination number of G, denoted by $\gamma_{\times 2}(G)$, is the minimum cardinality of a double dominating set of G. Chellali and Haynes [2] showed that if G is a claw-free graph without isolated vertices, then the paired domination number of G is at most the double domination number of G. In this paper, we show that if G is a H-free graph for some $H \in \{P_5, 2K_2 \cup K_1, \text{fork}\}$ without isolated vertices, then $\gamma_{pr}(G) \leq \gamma_{\times 2}(G)$.

References

[1] M. Blidia, M. Chellali, T. W. Haynes, Characterizations of trees with equal paired and double domination numbers. *Discrete Math.* **306** (2006), 1840–1845.

[2] M. Chellali and T. W. Haynes, On paired and double domination in graphs. *Util. Math.* **67** (2005), 161–71.

Perfect (1,2)-Dominating Sets in Graphs with a Few Large-Degree Vertices

M. Pirga(1)

(1) Rzeszow University of Technology, Poland

Let $k \geq 1$ be an integer. A subset $D \subseteq V(G)$ is a $(1,k)$-dominating set if for every vertex $v \in V(G) \setminus D$ there exist $u, w \in D$ such that $uv \in E(G)$ and $d_G(v,w) \leq k$. The concept of $(1,2)$-dominating sets was introduced in [1] and further studied in [2, 3], where A. Michalski defined a *proper* $(1,2)$-*dominating set* as a $(1,2)$-dominating set that is not $(1,1)$-dominating. Based on this idea, in [4] we introduced a *perfect* $(1,2)$-*dominating set* (shortly $(1,2)$-PDS) as a $(1,2)$-dominating set in which every vertex outside D is adjacent to exactly one vertex of D.

In this talk, we investigate the existence of $(1,2)$-PDS in graphs containing at most two vertices of maximum degree. In particular, we provide a complete characterization for the cases $\Delta(G) = n-1$ and $\Delta(G) = n-2$.

References

[1] S. M. Hedetniemi, S. T. Hedetniemi, J. Knisely, D. F. Rall, *Secondary domination in graphs*, AKCE International Journal of Graphs and Combinatorics, 5, (2008), 103–115.

[2] A. Michalski, *Secondary dominating sets in graphs and their modification*, Book of abstracts, The 7th Gdańsk Workshop on Graph Theory, 2019.

[3] A. Michalski, I. Włoch, M. Dettlaff, M. Lemańska, *On proper (1,2)-dominating sets*, Mathematical Methods in the Applied Sciences, 45(11), (2022), 7050–7057.

[4] U. Bednarz, M. Pirga, *(1, 2)-PDS in graphs with the small number of vertices of large degrees*, Opuscula Mathematica 45, no. 1 (2025), 53-62.

Disjoint dominating and 2-dominating sets in graphs: Hardness and Approximation results

Soumyashree Rana(1), Sounaka Mishra(2), B. S. Panda(1)

(1) Department of Mathematics, Indian Institute of Technology Delhi, India
(2) Department of Mathematics, Indian Institute of Technology Madras, India

A set $D \subseteq V$ of a graph $G = (V, E)$ is a dominating set of G if each vertex $v \in V \setminus D$ is adjacent to at least one vertex in D, whereas a set $D_2 \subseteq V$ is a 2-dominating (double dominating) set of G if each vertex $v \in V \setminus D_2$ is adjacent to at least two vertices in D_2. A graph G is a DD_2-graph if there exists a pair (D, D_2) of disjoint dominating set and 2-dominating set of G. In [1], several open problems related to DD_2-graphs were posed. In this paper, we answers some of these problems and present the following results: we provide an approximation algorithm for the problem of determining a minimal spanning DD_2-graph of minimum size (Min-DD_2) with an approximation ratio of 3; a minimal spanning DD_2-graph of maximum size (Max-DD_2) with an approximation ratio of 3; and the smallest number of edges which when added to a non-DD_2-graph results in a minimal spanning DD_2-graph for any graph (Min-to-DD_2) with an $O(\log n)$ approximation ratio. Additionally, we prove that Min-DD_2 and Max-DD_2 are APX-complete for graphs with maximum degree 4. Moreover, for a 3-regular graph, we show that Min-DD_2 and Max-DD_2 are approximable within a factor of 1.8 and 1.5 respectively.

References

[1] M.Miotk, J.Topp, and P.Żyliński. *Disjoint dominating and 2-dominating sets in graphs*, Discrete Optimization, 35:100553, (2020).

Induced cycles vertex number vs. $(1,2)$-domination in cubic graphs

Rija Erveš[(1)], Aleksandra Tepeh[(2,3)]

[(1)] University of Maribor, FCETEA, Slovenia
[(2)] University of Maribor, FEECS, Slovenia
[(3)] Institute of Mathematics, Physics and Mechanics, Slovenia

A $(1,2)$-dominating set in a graph G is a set S such that every vertex outside S has at least one neighbor in S, and each vertex in S has at least two neighbors in S. The $(1,2)$-domination number, $\gamma_{1,2}(G)$, is the minimum size of such a set, while $c_{\text{ind}}(G)$ is the cardinality of the largest vertex set in G that induces one or more cycles. In this paper, we initiate the study of a relationship between these two concepts and discuss how establishing such a connection can contribute to solving a conjecture on the lower bound of $c_{\text{ind}}(G)$ for cubic graphs. We also establish an upper bound on $c_{\text{ind}}(G)$ for cubic graphs and characterize graphs that achieve this bound.

References

[1] R. Erveš, A. Tepeh , Induced cycles vertex number and $(1,2)$-domination in cubic graphs, *Applied Math. Comput.* (to appear).

Outdegree conditions forcing directed cycles

A. Grzesik(1), J. Jaworska(1), P. Kuc(1)

(1) Jagiellonian University, Kraków, Poland

In 2010, Kelly, Kühn and Osthus [2] made a conjecture on the minimum semidegree which forces an oriented graph to contain a directed cycle of a given length at least 4. The conjecture was proven by its authors for cycles of length not divisible by 3 and in [1] for other cycles. We consider an analogous problem, but without the assumption on the minimum indegree, and prove an optimal bound on the minimum outdegree which forces an oriented graph to contain a directed cycle of a given large enough length.

References

[1] A. Grzesik, J. Volec, Degree conditions forcing directed cycles, *Int. Math. Res. Not. IMRN* 2023 (2023), 9711–9753.

[2] L. Kelly, D. Kühn, D. Osthus, Cycles of given length in oriented graphs, *J. Combin. Theory Ser. B* 100 (2010), 251–264.

Largest planar graphs of diameter 3 and fixed maximum degree

Antoine Dailly(1), Sasha Darmon(2), Ugo Giocanti(3), Claire Hilaire(4), Petru Valicov(5)

(1) Université Clermont Auvergne, INRAE, France
(2) LBBE, Université Lyon 1, & INRIA, Université Lyon 1, France
(3) Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, Poland
(4) FAMNIT, University of Primorska, Koper, Slovenia
(5) LIRMM, Université de Montpellier, CNRS, Montpellier, France

The degree diameter problem asks for the maximum possible number of vertices in a graph of maximum degree Δ and diameter D. In this paper, we focus on planar graphs of diameter 3. Fellows, Hell and Seyffarth [1] proved that for all $\Delta \geq 8$, the maximum number $\mathrm{np}_{\Delta,D}$ of vertices of a planar graph with maximum degree at most Δ and diameter at most 3 satisfies $\frac{9}{2}\Delta - 3 \leq \mathrm{np}_{\Delta,3} \leq 8\Delta + 12$. We show that the given lower bound is tight up to an additive constant, by proving that there exists a constant $c > 0$ such that $\mathrm{np}_{\Delta,3} \leq \frac{9}{2}\Delta + c$ for every $\Delta \geq 0$. Our proof consists in a reduction to the fractional maximum matching problem on a specific class of planar graphs, for which we show that the optimal solution is $\frac{9}{2}$, and characterize all graphs attaining this bound.

References

[1] Michael R. Fellows, Pavol Hell, Karen Seyffarth Large planar graphs with given diameter and maximum degree, *Discrete Applied Mathematics* 1995 61(2) pp.133-153.

Extremal problems on planar graphs

E. Győri(1)

(1) Renyi Institute of Mathematics, Budapest, Hungary

Let $\mathrm{ex}_{\mathcal{P}}(n, T, H)$ denote the maximum number of copies of T in an n-vertex planar graph which does not contain H as a subgraph. When $T = K_2$, $\mathrm{ex}_{\mathcal{P}}(n, T, H)$ is the planar Turán number of H, denoted by $\mathrm{ex}_{\mathcal{P}}(n, H)$. The topic of extremal planar graphs was initiated by Dowden (2016) [1]. He obtained sharp upper bound for both $\mathrm{ex}_{\mathcal{P}}(n, C_4)$ and $\mathrm{ex}_{\mathcal{P}}(n, C_5)$. In [2], we gave a sharp upper bound $\mathrm{ex}_{\mathcal{P}}(n, C_6) \leq \frac{5}{2}n - 7$, for all $n \geq 18$. We also pose a conjecture on $\mathrm{ex}_{\mathcal{P}}(n, C_k)$, for $k \geq 7$.

We [3] proved that for every integer $n \geq 6$, $\mathrm{ex}_{\mathcal{P}}(n, C_5, \emptyset)$ is $2n^2 - 10n + 12 + \mathbb{1}_{n=7}$.

And (see [4]) for every fixed $k \geq 3$, $\mathrm{ex}_{\mathcal{P}}(n, C_{2k}, \emptyset)$ is $n^k/k^k + o(n^k)$. In this lecture, we present more recent similar results related to cycles and paths.

References

[1] C. Dowden. Extremal C_4-free/C_5-free planar graphs. Journal of Graph Theory 83 (2016), 213-230.

[2] D. Ghosh, E. Győri, R. R. Martin, A. Paulos, C. Xiao. Planar Turán Number of the 6-Cycle. SIAM Journal on Discrete Mathematics 36 (3) (2022), 2028-2050.

[3] E. Győri, A. Paulos, N. Salia, C. Tompkins, O. Zamora. The maximum number of pentagons in a planar graph. J. Graph Th., 108 (2025). pp. 229-256.

[4] Z. Lv, E. Győri, Z. He, N. Salia, C. Tompkins, X. Zhu. The maximum number of copies of an even cycle in a planar graph. J. Combinatorial Th. B, 167 (2024) 15-22.

On the Turán number of the expansion of the t-fan

X. Cheng[(1)], D. Gerbner[(2)], H. Hama Karim[(3)], J. Zhou[(4)]

[(1)] Northwestern Polytechnical University, Shaanxi, China.
[(2)] Alfréd Rényi Institute of Mathematics, Budapest, Hungary.
[(3)] Budapest University of Technology and Economics, Budapest, Hungary.
[(4)] Shanghai University, Shanghai, China.

The t-fan, F_t, is the graph on $2t+1$ vertices consisting of t triangles that intersect at exactly one common vertex. For a given graph F, the r-expansion F^r of F is the r-uniform hypergraph obtained from F by adding $r-2$ distinct new vertices to each edge of F. The Turán number of an r-uniform hypergraph $\mathcal{H}$, $ex_r(n, \mathcal{H})$, is the maximum number of hyperedges an r-uniform n-vertex hypergraph can have without containing $\mathcal{H}$ as a subhypergraph. We determine the Turán number of the 3-expansion of the t-fan for sufficiently large n. Namely, we show that $\mathrm{ex}_3(n, \mathcal{F}_t^3) = \binom{n}{3} - \binom{n-t}{3}$.

Unavoidable subgraphs in digraphs with large out-degrees

Tomáš Hons(1), Tereza Klimošová(2), Gaurav Kucheriya(2), David Mikšaník(1), Josef Tkadlec(1), Mykhaylo Tyomkyn(2)

(1) Computer Science Institute (IÚUK, MFF), Charles University, Prague, Czech Republic
(2) Department of Applied Mathematics (KAM, MFF), Charles University, Prague, Czech Republic

We ask the question, which oriented trees T must be contained as subgraphs in every finite directed graph of sufficiently large minimum out-degree. We formulate the following simple condition: all vertices in T of in-degree at least 2 must be on the same 'level' in the natural height function of T. We prove this condition to be necessary and conjecture it to be sufficient. In support of our conjecture, we prove it for a fairly general class of trees.

An essential tool in the latter proof, and a question interesting in its own right, is finding large subdivided in-stars in a directed graph of large minimum out-degree. We conjecture that any digraph and oriented graph of minimum out-degree at least $k\ell$ and $k\ell/2$, respectively, contains the $(k-1)$-subdivision of the in-star with ℓ leaves as a subgraph; this would be tight and generalizes a conjecture of Thomassé. We prove this for digraphs and $k = 2$ up to a factor of less than 4.

Random embeddings of bounded degree trees with optimal spread

P. Bastide(1), C. Legrand-Duchesne(2), A. Müyesser(1)

(1) University of Oxford
(2) Jagiellonian Univertisity, Kraków

A seminal result of Komlós, Sárközy, and Szemerédi [1] states that any n-vertex graph G with minimum degree at least $(1/2+\alpha)n$ contains every n-vertex tree T of bounded degree. Recently, Pham, Sah, Sawhney, and Simkin [2] extended this result to show that such graphs G in fact support an optimally spread distribution on copies of a given T, which implies, using the recent breakthroughs on the Kahn-Kalai conjecture, the robustness result that T is a subgraph of sparse random subgraphs of G as well. Pham, Sah, Sawhney, and Simkin construct their optimally spread distribution by following closely the original proof of the Komlós-Sárközy-Szemerédi theorem which uses the blow-up lemma and the Szemerédi regularity lemma. We give an alternative, regularity-free construction that instead uses the Komlós-Sárközy-Szemerédi theorem (which has a regularity-free proof due to Kathapurkar and Montgomery) as a black-box. Our proof is based on the simple and general insight that, if G has linear minimum degree, almost all constant sized subgraphs of G inherit the same minimum degree condition that G has.

References

[1] J. Komlós, G. Sárközy, and E. Szemerédi, Proof of a packing conjecture of Bollobás, *Combinatorics, Probability and Computing* vol.4 1995 pp. 241–255.

[2] H. Pham, A. Sah, M. Sawhney, and M. Simkin, A toolkit for robust thresholds, *arXiv:2210.03064 [math.CO]*, 2022.

Constructions of Turán systems that are tight up to a multiplicative constant

O. Pikhurko(1)

(1) Mathematics Institute and DIMAP, University of Warwick, Coventry CV4 7AL, UK

The Turán density $t(s,r)$ is the asymptotically smallest edge density of an r-graph for which every set of s vertices contains at least one edge. The question of estimating this function received a lot of attention since it was first raised by Turán in 1941. A trivial lower bound is $t(s,r) \geq 1/\binom{s}{s-r}$. In the early 1990s, de Caen [1] conjectured that $t(r+1,r)$ grows faster than $O(1/r)$ and offered 500 Canadian dollars for resolving this question.

I will give an overview of this area and present a construction from [2] disproving this conjecture by showing more strongly that for every integer R there is C such that $t(r+R,r) \leq C/\binom{r+R}{R}$, that is, the trivial lower bound is tight for every fixed R up to a multiplicative constant $C = C(R)$.

References

[1] D. de Caen. The current status of Turán's problem on hypergraphs. In *Extremal problems for finite sets (Visegrád, 1991)*, volume 3 of *Bolyai Soc. Math. Stud.*, pages 187–197. János Bolyai Math. Soc., Budapest, 1994.

[2] O. Pikhurko, *Constructions of Turán systems that are tight up to a multiplicative constant*, Advances in Mathematics **464** (2025), 110148, 11 pages.

Distinct degrees and homogeneous sets in graphs

Eoin Long[2], Laurentiu Ploscaru[1]

[1] Alfred Rényi Institute of Mathematics, Budapest, Hungary
[2] University of Birmingham, UK

In this talk we investigate the extremal relationship between the order of a largest homogeneous set (clique or independent set) in a graph G and the maximal number of distinct degrees that appear in an induced subgraph of G, denoted by $\hom(G)$ and $f(G)$ respectively. This topic has been well studied by several researchers over the last 40 years, beginning with Erdős, Faudree and Sós in the regime when $\hom(G) = O\big(\log|G|\big)$.

Our main theorem asymptotically settles this question, improving on multiple earlier estimates. More precisely, we show that any n-vertex graph G satisfies:

$$f(G) \geq \min\left(\sqrt[3]{\frac{n^2}{\hom(G)}}\ ,\ \frac{n}{\hom(G)}\right) \cdot n^{-o(1)}.$$

This relationship is tight (up to the $n^{o(1)}$ term) for all possible values of $\hom(G)$, as demonstrated by appropriately generated Erdős−Renyi random graphs. Our approach to lower bounding $f(G)$ proceeds via a translation into an (almost) equivalent probabilistic problem, which is effective for arbitrary graphs.

Cycle lengths in graphs of given minimum degree

Yandong Bai(1), Andrzej Grzesik(2), Binlong Li(1),
Magdalena Prorok(3)

(1) Northwestern Polytechnical University, Xi'an, China
(2) Jagiellonian University, Kraków, Poland
(3) AGH University, Kraków, Poland

We present minimum degree conditions which forces a graph to contain a cycle of length ℓ modulo k for fixed k and ℓ. Our outcomes improve the results of Gao, Huo, Liu and Ma [1]. Consequently, we determine the maximum number of edges in a graph that does not contain a cycle of length 0 modulo k for odd k.

References

[1] J. Gao, Q. Huo, C. Liu, J. Ma, A unified proof of conjectures on cycle lengths in graphs, *International Mathematics Research Notices* (2022) (10) 7615–7653.

Antidirected paths in oriented graphs

Andrzej Grzesik[(1)], Marek Skrzypczyk[(1)]

[(1)] Jagiellonian University, Cracow, Poland

We show that for any integer $k \geq 4$, every oriented graph with minimum semidegree bigger than $\frac{1}{2}(k-1+\sqrt{k-3})$ contains an antidirected path of length k. Consequently, every oriented graph on n vertices with more than $(k-1+\sqrt{k-3})n$ edges contains an antidirected path of length k. This asymptotically proves the antidirected path version of a conjecture of Stein and of a conjecture of Addario-Berry, Havet, Linhares Sales, Reed and Thomassé, respectively.

References

[1] A.Grzesik, M.Skrzypczyk, Antidirected paths in oriented graphs, *arXiv:2506.11866*, 2025.

Gauss words and rhythmic canons

Jarosław Grytczuk[1]

[1] Warsaw University of Technology, Warsaw, Poland

A *rhythmic canon* is a combinatorial structure consisting of repeating copies of the same *motif.* These copies may be variously transformed and their placement on the time-line can also be quite arbitrary. Thus, a purely abstract rhythmic canon can be identified with an *ordered* hypergraph on the set of vertices $\{1, 2, \ldots, n\}$ whose edges correspond to the transformed copies of the leading motif. If the edges partition the set of vertices (a hypergraph is a *perfect matching*), then it is convenient to represent the canon by a *word* with same letters occupying positions of a fixed copy of the motif. If all copies are of the same size (a hypergraph is uniform), then each letter in the word occurs the same number of times. Words with this property are called *Gauss words*, in honor of the researcher who first used them in studying self-crossing curves on the plane.

There are many exciting problems about rhythmic canons. I will present a few of them during the talk. To get a foretaste, consider the following puzzle invented by Tom Johnson, a composer. Take a look at the word

$$ABCDCBCADBEEEDA.$$

It is an example of a *perfect rhythmic canon* $K(5, 3)$, that is, a *tiling* of the interval into five 3-term arithmetic progressions, each with a distinct gap. There are no such canons with two, three, four or six progressions, but it is known that $K(n, 3)$ exist for all $7 \leqslant n \leqslant 19$. In particular, there are 9257051746 different canons $K(19, 3)$. Is it true that for every $n \geqslant 7$ there is at least one perfect rhythmic canon $K(n, 3)$? Perfect canons $K(n, 4)$, built of 4-term arithmetic progressions of pairwise different gaps, are known to exist for all $15 \leqslant n \leqslant 23$. In particular, there are 19490 different canons $K(23,4)$. What about canons $K(n,r)$ with $r \geqslant 5$? Do they exist for every fixed r and arbitrarily large n?

On b-acyclic chromatic number of cubic and subcubic graphs

M. Anholcer(1), S. Cichacz(2), I. Peterin(3)

(1) Poznań University of Economics and Business, Poznań, Poland
(2) AGH University of Krakow, Cracow, Poland
(3) University of Maribor, Maribor, Slovenia

Let G be a graph. An acyclic k-coloring of G is a map $c : V(G) \to \{1, \ldots, k\}$ such that $c(u) \neq c(v)$ for any $uv \in E(G)$ and the subgraph induced by the vertices of any two colors $i, j \in \{1, \ldots, k\}$ is a forest. If every vertex v of a color class V_i misses a color $\ell_v \in \{1, \ldots, k\}$ in its closed neighborhood, then every $v \in V_i$ can be recolored with ℓ_v and we obtain a $(k-1)$-coloring of G. If a new coloring c' is also acyclic, then such a recoloring is an acyclic recoloring step and c' is in relation $\triangleleft_a$ with c. The acyclic b-chromatic number $A_b(G)$ of G is the maximum number of colors in an acyclic coloring where no acyclic recoloring step is possible. Equivalently, it is the maximum number of colors in a minimum element of the transitive closure of $\triangleleft_a$. In this talk, we develop the results presented in [1] by considering $A_b(G)$ of cubic and subcubic graphs.

References

[1] M. Anholcer, S. Cichacz, I. Peterin, On b-acyclic chromatic number of a graph. Comp. Appl. Math. 42, 21 (2023).

First-Fit Coloring of Forests in Random Arrival Model

B. Bosek(1,2), G. Gutowski(1), M. Lasoń(3), J. Przybyło(4)

(1) Institute of Theoretical Computer Science, Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, Poland
(2) Institute of Informatics, University of Warsaw, Poland
(3) Institute of Mathematics of the Polish Academy of Sciences, Warszawa, Poland
(4) AGH University of Krakow, Faculty of Applied Mathematics, Kraków, Poland

We study the performance of the First-Fit coloring algorithm on forests in the random arrival model. While this algorithm is known to use $\Theta(\log n)$ colors in the worst-case (adversarial) online model, its average-case performance under a random vertex permutation has been less understood.

We close this gap by providing tight asymptotic bounds. We show that for any forest with n vertices, the expected number of colors used by First-Fit is at most $(1 + o(1))\frac{\ln n}{\ln \ln n}$. Furthermore, we prove this bound is optimal by constructing a family of forests that requires $(1 - o(1))\frac{\ln n}{\ln \ln n}$ colors in expectation. Our result precisely characterizes the performance of First-Fit for this graph class, showing a modest but significant gain over the adversarial setting.

References

[1] B.Bosek, G.Gutowski, M.Lasoń, J.Przybylo, First-Fit Coloring of Forests in Random Arrival Model. *MFCS 2024*, 33:1-33:10.

On the distinguishing chromatic number in hereditary graph classes

C. Brause[(1,2)], R. Kalinowski[(2)], M. Pilśniak[(2)], I. Schiermeyer[(1,2)]

[(1)] TU Bergakademie Freiberg, Freiberg, Germany
[(2)] AGH University of Krakow, Kraków, Poland

The distinguishing chromatic number of a graph G, denoted by $\chi_D(G)$, is the minimum number of colours in a proper vertex colouring of G that is preserved by the identity automorphism only. Collins and Trenk proved $\chi_D(G) \leq 2\Delta(G)$ for any connected graph G, and that equality holds for complete balanced bipartite graphs $K_{p,p}$ and for C_6.

In this talk, we show that the upper bound on $\chi_D(G)$ can be substantially reduced if we forbid some small graphs as induced subgraphs of G, that is, we study the distinguishing chromatic number in some hereditary graph classes.

Packing List-Colorings and the Proper Connection Number of Connected Graphs

Sebastian Czerwiński[(1)]

[(1)] University of Zielona Góra, Institute of Mathematics, Poland

The *proper connection number* of a connected graph G is the minimum number of colors t required for a proper connected t-coloring of G; that is, an edge coloring of G such that between every pair of distinct vertices there exists a properly colored path.

We also consider list and list-packing versions of this number. Given a list L-edge-assignment of G, with $|L| = k$, an *L-packing proper connected coloring* of G is a collection of k mutually disjoint proper connected colorings $c_1, c_2, \ldots, c_k$ of the edges of G; that is, these colorings satisfy the conditions that for every vertex $v \in V(G)$ we have $c_i(v) \in L(v)$, and $c_i(v) \neq c_j(v)$ whenever $i \neq j$.

We discuss the origin of list-packing colorings and provide new results on this topic.

Interval colouring of oriented graphs

Ewa Drgas-Burchardt[(1)]

[(1)] University of Zielona Góra, Zielona Góra, Poland

An oriented graph is *interval colourable* if it admits an arc colouring with integers such that, for every vertex, the integers assigned to the in-arcs incident to this vertex are pairwise distinct, the integers assigned to the out-arcs incident to this vertex are also pairwise distinct, and both of these sets form intervals of integers. Since there exist oriented graphs that are not interval colourable, we analise the *interval colouring reorientation number* of an oriented graph D, denoted by $icr(D)$, defined as the minimum number of arcs of D that should be reversed so that a resulting oriented graph is interval colourable.

In this talk, we present properties and constraints of the interval colouring reorientation number, as well as its connections to other well-known parameters studied in the theory of graphs and digraphs.

References

[1] M.Borowiecka-Olszewska, E.Drgas-Burchardt, R.Zuazua, On interval colouring reorientation number of oriented graphs, *Discrete Appl. Math.* 2025 pp.65-80.

Totally Locally Irregular Decompositions of Graphs

Anna Flaszczyńska(1), Aleksandra Gorzkowska(1), Igor Grzelec(1), Alfréd Onderko(2), Mariusz Woźniak(1)

(1) AGH University, Cracow, Poland
(2) P.J. Šafárik University, Košice, Slovakia

A locally irregular graph is a graph in which all adjacent vertices have distinct degrees. In article [1], the authors described the minimum number of locally irregular subgraphs into which a graph can be decomposed. This can be viewed as a graph coloring, where each color corresponds to a locally irregular subgraph. In [1], a total version of this problem is also defined.

In the problem of totally locally irregular decomposition of graphs, we aim to find the minimum number of colors in a total coloring of the graph such that, within each color class, all adjacent vertices have distinct total degrees.

References

[1] O. Baudon, J. Bensmail, J. Przybyło, M. Woźniak, On decomposing regular graphs into locally irregular subgraphs, *European Journal of Combinatorics*, 2015, 49, pp. 90–104.

Packing coloring of graphs with long paths

H. Furmańczyk[(1)], D. Gözüpek[(2)], S. Özkan[(2)]

[(1)] University of Gdańsk, Gdańsk, Poland
[(2)] Gebze Technical University, Gebze, Turkey

A *packing coloring* of a graph G is a mapping $c : V(G) \to \mathbb{N}$ such that any two distinct vertices assigned color i are at distance greater than i in G. This generalizes classical proper coloring by incorporating distance constraints that grow with the color index. The smallest integer k for which such a coloring exists using colors $1, \ldots, k$ is called the *packing chromatic number*, denoted $\chi_p(G)$.

We define a new class of graphs called *path-aligned graph products*, denoted by $P_n \Diamond_l G$. Let n and l be positive integers such that $l \mid n$, and let G be a connected vertex-transitive graph that contains a path P_l as a subgraph.The graph $P_n \Diamond_l G$ is constructed as follows.

- Start with the path P_n, with vertex sequence $v_1, v_2, \ldots, v_n$.
- Partition P_n into n/l consecutive, disjoint subpaths of length l, i.e., the i-th subpath is $P_l^{(i)} = (v_{(i-1)l+1}, \ldots, v_{il})$ for $i = 1, 2, \ldots, n/l$.
- For each i, take a copy $G^{(i)}$ of the graph G, and identify the subpath $P_l^{(i)} \subseteq P_n$ with a fixed copy of $P_l \subseteq G^{(i)}$. That is, the vertices of $P_l^{(i)}$ are merged with the corresponding vertices of the embedded path P_l in $G^{(i)}$.

We investigate the packing chromatic number χ_p of such constructions for various choices of G, including cycles and complete graphs, and determine exact values or bounds in these cases. Furthermore, we extend our results to selected classes of corona products, including generalized coronas, which share similar alignment properties.

Odd Coloring: Complexity and Algorithms

Rumki Ghosh[(1)], B. S. Panda[(1)]

[(1)] Department of Mathematics, Indian Institute of Technology Delhi, Hauz Khas, New Delhi 110016, INDIA

An odd k-coloring of a graph $G = (V, E)$ is a proper k-coloring of G such that for every non-isolated vertex $v \in V$, there exists at least one color that appears an odd number of times in the open neighborhood of v. The minimum k for which G admits an odd k-coloring is called the odd chromatic number of G and is denoted by $\chi_o(G)$. Given a graph G and a positive integer k, Decide Odd Coloring Problem is to decide whether G admits an odd k-coloring. Decide Odd Coloring Problem is known to be NP-complete for general graphs [1]. In this paper, we strengthen this hardness result by proving that Decide Odd Coloring Problem remains NP complete for dually chordal graphs. On the positive side, we prove that for any proper interval graph G, the odd chromatic number satisfies $\omega(G) \leq \chi_o(G) \leq \omega(G) + 1$. We further characterize the proper interval graphs for which $\chi_o(G) = \omega(G)$, and those for which $\chi_o(G) = \omega(G) + 1$. We present a linear-time algorithm to compute the odd chromatic number of block graphs. Finally, we prove that the odd chromatic number of an interval graph G is either $\omega(G)$ or $\omega(G)+1$. Further, we characterize the interval graphs having $\chi_o(G) = \omega(G)$ and $\chi_o(G) = \omega(G) + 1$.

References

[1] Yair Caro, Mirko Petruševski, Riste Škrekovski, Remarks on Odd Coloring of Graphs, *Discrete Applied Mathematics*, 321: 392-401, 2022.

Computational and algebraic approaches to open XOR-magic graphs

Sylwia Cichacz[(1)], Hubert Grochowski[(2)], Rita Zuazua[(3)]

(1) AGH University of Kraków, Kraków, Poland

(2) Warsaw University of Technology, Warsaw, Poland

(3) Universidad Nacional Autónoma de México, Mexico

A graph $G = (V, E)$ with $|V| = 2^n$ is called *(open) XOR-magic graph*, if it is connected and there exists a bijective labeling $\ell : V \to (\mathbb{Z}_2)^n$ such that for each vertex $v \in V$, sum of labels over (open) closed neighborhood of v is equal to $\mathbf{0}$. This labeling is a special case of group distance magic labeling of graphs.

Batal posed the following open problem: does it exist any even regular XOR-magic or odd regular open XOR-magic graph? In this talk, we will present positive answers to these questions, as well as a generalization about the existence of such graphs of order 2^n for each $n \geq 4$. Furthermore, we will present obtained algebraic approach to non-existence of open XOR-magic labelings and its application to various classes of circulant graphs.

Hubert Grochowski's research was funded by the Warsaw University of Technology within the Excellence Initiative: Research University (IDUB) programme.

References

[1] D. Froncek, *Group distance magic labeling of of Cartesian products of cycles*, Australasian Journal of Combinatorics 55 (2013) 167–174.

[2] M. Anholcer, S. Cichacz, D. Froncek, R. Simanjuntak, J. Qiu, *Group distance magic and antimagic hypercubes*, Discrete Mathematics 344 (2021) 112625.

[3] A. Batal, *On the construction of xor-magic graphs*, preprint.

Weak and strong local irregularity of digraphs

I. Grzelec(1), A. Onderko(2), M. Woźniak(1)

(1) AGH University of Krakow, Kraków, Poland
(2) P.J. Šafárik University, Košice, Slovakia

Local Irregularity Conjecture states that every connected graph, except special cacti, can be decomposed into at most three *locally irregular graphs*, i.e., graphs in which adjacent vertices have different degrees [1, 2]. The notion of local irregularity was defined for digraphs in several different ways. At the beginning of this talk we present the already known concepts of local irregularity for digraphs with motivations, main conjectures and known results. Then we introduce the following new methods of defining a *locally irregular digraph*. The first one, *weak local irregularity*, is based on distinguishing adjacent vertices by *indegree-outdegree pairs*, and the second one, *strong local irregularity*, asks for *different balanced degrees* (i.e., difference between the outdegree and the indegree of a vertex) of adjacent vertices. For both of these irregularities, we define locally irregular decompositions and colorings of digraphs. We also provide related conjectures on the minimum number of colors in weak and strong locally irregular colorings and support them with new results for various classes of digraphs.

References

[1] O. Baudon, J. Bensmail, J. Przybyło, M. Woźniak, On decomposing regular graphs into locally irregular subgraphs, *Europ. J. Combin.*, **49** (2015), 90–104.

[2] J. Sedlar, R. Škrekovski, Local Irregularity Conjecture vs. cacti, *Discrete Appl. Math.* **343** (2024), 115–133.

Majority Additive Coloring

Mateusz Kamyczura(1)

(1) AGH University, Cracow, Poland

Majority additive coloring is a type of coloring where each vertex is assigned a number, and the sum of its neighbors' numbers, called the neighbor sum, is then computed. For the coloring to be valid, in the neighborhood of each vertex, at most half of its neighbors can share the same neighbor sum. Therefore, majority additive coloring is a combination of two known problems: additive coloring and majority coloring. The majority additive chromatic number, denoted by $\chi_{mac}(G)$, is the smallest number of colors required to achieve a majority additive coloring of G. We present several results regarding χ_{mac} for different types of graphs. For complete graphs and cycles, we have determined the exact value of the parameter, while for trees, we have found a tight upper bound. The main result of this work shows that for graphs with girth greater than 5, a sufficiently large maximum degree, and a minimum degree close to the maximum degree, it is sufficient to use only the numbers 1 and 2.

Odd coloring of k-trees

M. Kashima[(1)], K. Ozeki[(2)]

[(1)] Keio University, Yokohama, Japan
[(2)] Yokohama National University, Yokohama, Japan

For a graph G, an odd coloring of G is a proper coloring φ such that every non-isolated vertex v has a color c such that $|\varphi^{-1}(c) \cap N(v)|$ is an odd integer. A graph is said to be odd k-colorable if it admits an odd coloring with at most k colors. This notion was introduced by Petruševski and Škrekovski [2] in 2022, where they investigated odd coloring of planar graphs.

In this talk, we focus on odd coloring of k-trees. For a positive integer k, a graph which is obtained from K_{k+1} by recursively adding a vertex which is joined to a clique of order k is called a k-tree. For any $k \geq 1$, it is easy to see that there are infinitely many k-trees that are not odd $(k+1)$-colorable. On the other hand, according to a result by Cranston et al. [1], it follows that every graph of tree-width at most k is odd $(2k+1)$-colorable, and hence every k-tree is odd $(2k+1)$-colorable. We improve this bound by showing that every k-tree is odd $(k + 2\lfloor \log_2 k \rfloor + 3)$-colorable. Furthermore, when $k = 2, 3$, we show that every 2-tree is odd 4-colorable and that every 3-tree is odd 5-colorable, both of which are tight bounds. In particular, since every maximal outerplanar graph is a 2-tree, this implies that every maximal outerplanar graph is odd 4-colorable.

References

[1] D. W. Cranston, M. Lafferty, Z.-X. Song, A note on odd colorings of 1-planar graphs, *Discrete Appl. Math.*, 2023 pp.112-117.

[2] M. Petruševski, R. Škrekovski, Colorings with neighborhood parity condition, *Discrete Appl. Math.* 2022 pp.385-391.

Line graph orientations and list edge colorings of regular graphs

G. Gutowski[(1)], J. Kozik[(1)], B. Podkanowicz[(1)]

[(1)] Jagiellonian University, Krakow, Poland

The List Edge Coloring Conjecture states that for any graph G, the list chromatic index $ch'(G)$ equals the chromatic index $\chi'(G)$. A major breakthrough toward resolving this conjecture was Galvin's proof that it holds for bipartite graphs. It is natural to consider extending his coloring procedure to general graphs by decomposing them into bipartite subgraphs. However, such decompositions turn out to be incompatible with the method.

In 1996, Kahn proved that the conjecture holds asymptotically, establishing an upper bound of $\chi'(G) + o(\chi'(G))$. A later refinement by Häggkvist and Janssen, yielding a bound of $\chi'(G)+\widetilde{O}(\chi'(G)^{2/3})$, relies on the Alon–Tarsi polynomial method. This approach derives bounds from the existence of specific orientations of the line graph. Interestingly, such orientations can be constructed from those of bipartite subgraphs arising from natural decompositions. Therefore, any improvement of the result for bipartite graphs could potentially enhance the general bounds. Unfortunately, no such bipartite-specific improvements are currently known.

In this work, we explore this approach and present partial results on coloring line graphs of complete multipartite graphs.

List distinguishing index of graphs

Jakub Kwaśny[(1)], Marcin Stawiski[(1)]

[(1)] AGH University of Krakow

An edge colouring of a graph is called distinguishing if there is no non-trivial automorphism which preserves it. Distinguishing colourings gained quite a lot of attention since 1990s, and are still extensively studied. The most notable recent result in this area is the confirmation by Babai of the Infinite Motion Conjecture proposed by Tucker.

The talk will be about the list variant of this problem. We will present a general bound of $\Delta(G)-1$ for all connected graphs apart from some classified exceptions. This bound is optimal and it matches the best known bound for non-list colourings.

Then, we will discuss an improvement of the result of Lehner, Pilśniak, and Stawiski, which states that there is a distinguishing 3-edge-colouring of any connected regular graph except K_2. We prove that every at most countable, finite or infinite, connected regular graph of order at least 7 admits a distinguishing edge colouring from any set of lists of length 2.

References

[1] J. Kwaśny, M. Stawiski, *Distinguishing regular graphs from lists*, arXiv:2207.14728.

[2] J. Kwaśny, M. Stawiski, *List distinguishing index of graphs*, arXiv:2306.06418.

On the Orbital Chromatic Polynomial

Klaus Dohmen[(1)], Mandy Lange-Geisler[(2)]

[(1)] Mittweida University of Applied Sciences, Mittweida, Germany
[(2)] Mittweida University of Applied Sciences, Mittweida, Germany

The orbital chromatic polynomial, introduced by Cameron and Kayibi in 2007, counts the number of proper λ-colorings of a graph modulo a group of symmetries. The polynomial has been investigated for specific graphs, including the Petersen graph, complete graphs, null graphs, paths, and cycles of small length. So far, no general formula for the orbital chromatic polynomial of the n-cycle for arbitrary n has been established.

In this talk we present such formula for the group of rotations and the full automorphism group of the n-cycle. As a side result, we obtain a new proof of Fermat's little theorem.

References

[1] K. Dohmen, M. Lange-Geisler, The Orbital Chromatic Polynomial of a Cycle, url: `https://arxiv.org/abs/2009.08235`.

Colouring cubic multipoles

R. Lukoťka[(1)]

[(1)] Comenius University, Bratislava, Slovakia

To study 3-edge-uncolourability of a cubic graph one can take a cut containing k edges and split the graph into two graph parts, called cubic k-poles. Each 3-edge-colouring of a k-pole induces a k-tuple of colours on the dangling edges, called boundary colouring. All colourings of the k-pole induce a (multi)set of boundary colourings, called colouring set. A colouring set contains only colourings satisfying parity lemma and the set has to be closed under Kempe switches. For $k \leq 5$ these two conditions are not only necessary but also sufficient. We will focus on the case where $k = 6$. We introduce a new equivalence relation that greatly reduces the number of colouring sets one needs to consider. We present the results of computational experiments using this equivalence relation.

For planar graphs Four Colour Theorem can be used to restrict colouring sets of k-poles. We show that certain generalised flow polynomials are an efficient tool to capture the number of colourings with given boundary. We explore conditions that Four Colour Theorem imposes on the polynomial and study k-poles that are close to "refuting" the Four Colour Theorem with respect to their polynomial coefficients.

Sufficient forbidden immersion conditions for graphs to be 7-colorable

S. Maezawa(1)

(1) Nihon University, Tokyo, Japan

A graph H is an *immersion* of a graph G if there exist an injective function $f_1 : V(H) \to V(G)$ and a mappign f_2 from the edges of H to paths of G satisfying that

- for $uv \in E(H)$, $f_2(uv)$ is a path connecting $f_1(u)$ and $f_1(v)$ and
- for edges $e, e' \in E(H)$ with $e \neq e'$, $f_2(e)$ and $f_2(e')$ are pairwise edge-disjoint.

In analogy with Hadwiger's conjecture, Abu-Khzam and Langston [1] proposed the follwoing conjecture : every graph with no K_t as an immersion is $(t-1)$-colorable. Lescure and Meyniel [2] proved the conjecture for $t = 5, 6$ and DeVos, Kawarabayashi, Mohar, and Okamura [3] proved the conjecture for $t = 7$. In this talk, we discuss the conjecture for $t = 8$.

References

[1] F. N. Abu-Khzam and M. A. Langston, Graph coloring and the immersion order, in: Computing and Combinatorics, in: Lecture Notes in Comput. Sci., vol. 2697. Springer, Berlin, 2003, pp. 394–403.

[2] F. Lescure and H. Meyniel, On a problem upon configurations contained in graphs with given chromatic number, Graph theory in memory of G. A. Dirac, 325–331, Ann. Discrete Math., 41, North-Holland, Amsterdam, 1989.

[3] M. DeVos, K. Kawarabayashi, B. Mohar, and H. Okamura, Immersing small complete graphs, *Ars. Math. Contemp.* **3** (2) (2010) 139–146.

Erdős-Pósa property of cycles that are far apart

Piotr Micek(1)

(1) Jagiellonian University, Kraków, Poland

We prove that there exist functions f and g such that for all nonnegative integers k and d, for every graph G, either G contains k cycles such that vertices of different cycles have distance greater than d in G, or there exists a subset X of vertices of G, with $|X| \leqslant f(k)$ such that $G - B_G(X, g(d))$ is a forest, where $B_G(X, r)$ denotes the set of vertices of G having distance at most r from a vertex of X.

Edge-uncoverability by four perfect matchings in cubic graphs

Makuochukwu Felix Oguagbaka(1), Robert Lukoťka(1)

(1) Comenius University, Bratislava, Slovakia

Several longstanding conjectures in graph theory, including the Cycle Double Cover Conjecture, can be reduced to the case of cubic graphs. A notable parameter in this context is the perfect matching index of a cubic graph, defined as the minimum number of perfect matchings needed to cover its edges. In particular, if these conjectures hold for cubic graphs with perfect matching index at least 5, they hold in general.

In this talk, we introduce several invariants that capture how far a cubic graph G is from being coverable by four perfect matchings. One such invariant is the *four perfect matching defect* of G, denoted by $d_{PM}(G)$, defined as the minimum number of edges of G not covered by four perfect matchings. Another is the *four matching cover defect* of G, denoted by $d_M(G)$, defined as the minimum sum of defects of matchings over all matching covers of G containing four matchings, where the *defect of a matching* is half of the number of vertices it leaves uncovered. We prove that $d_M(G) \leq d_{PM}(G)$. Furthermore, we show that for each integer k, there exists a cubic graph with $d_M(G) = d_{PM}(G) = k$; that is, there are cubic graphs that are far from being coverable by four perfect matchings. We also present another family of cubic graphs, in which, for each integer k, there exists a cubic graph with $2d_M(G) \leq d_{PM}(G) = 2k$.

Beyond offering new perspectives on the structure of cubic graphs, we also demonstrate that such measures yield partial results toward resolving these famous conjectures.

List extensions of majority edge colourings

P. Pękała[(1)], J. Przybyło[(1)]

[(1)] AGH University, Cracow, Poland

A *majority edge colouring* of a graph G is a colouring of the edges of G such that for each vertex v of G, at most half the edges incident with v have the same colour. More generally, for a natural number $k \geq 2$, a $1/k$-*majority edge-colouring* of a graph is a colouring of the edges of G such that for every colour c and every vertex v of G at most $1/k$ of the edges incident with v have the colour c. This notion was introduced in 2023 by Bock, Kalinowski, Pardey, Pilśniak, Rautenbach and Woźniak [1].

We investigate possible list extensions of generalised majority edge colourings. In particular, given a graph G, a list assignment L and a *majority tolerance* $\alpha \in (0, 1)$, an α-*majority* L-*colouring* of G is a colouring $\omega : E \to C$ from the given lists such that for every $v \in V$ and each $c \in C$, the number of edges coloured c which are incident with v does not exceed $\alpha \cdot d(v)$. We discuss some restrictions necessary to extend this notion to a more general setting with diversified $\alpha = \alpha(c)$ majority tolerances for distinct colours $c \in C$. In particular, for any list assignment $L : E \to 2^C$ with $\sum_{c \in L(e)} \alpha(c) \geq 1 + \varepsilon$ and $|L(e)| \leq \ell$ for each edge e, we show that there exists an α-majority L-colouring of G, provided that $\delta(G) = \Omega(\ell^2 \varepsilon^{-2})$.

References

[1] F. Bock, R. Kalinowski, J. Pardey, M. Pilśniak, D. Rautenbach, M. Woźniak, Majority Edge-Colorings of Graphs, *Electron. J. Combin.* 30(1) (2023), #P1.42.

[2] P. Pękała, J. Przybyło, On Generalised Majority Edge-Colourings of Graphs, *Electron. J. Combin.* 31(4) (2024), #P.4.66.

[3] P. Pękała, J. Przybyło, On list extensions of the majority edge colourings, arXiv:2502.12688 (2025).

Soft Happy Colouring

Mohammad H. Shekarriz[(1)]

[(1)] School of Information Technology, Deakin University, Burwood 3125, VIC, Australia

For a coloured graph G and $0 \leq \rho \leq 1$, a vertex v is ρ-happy if at least $\rho \deg(v)$ of its neighbours share its colour. The soft happy colouring problem seeks a colouring σ that extends a given precolouring and maximises the number of ρ-happy vertices [3]. This NP-hard problem is closely linked to community detection in graphs. For example, for a graph in the stochastic block model (SBM) and for suitable ρ, with high probability, complete soft happy colourings can be achieved by the planted community structure [1]. Moreover, for $0 \leq \rho_1 < \rho_2 \leq 1$, complete ρ_2-happy colourings achieve higher detection accuracy than complete ρ_1-happy colourings, and when ρ surpasses a critical threshold, it is unlikely to find a complete ρ-happy colouring with near-equal class sizes [2]. Finally, we survey existing algorithms and propose novel heuristic, local search, evolutionary, metaheuristic, and matheuristic approaches that enhance solution quality for soft happy colouring.

References

[1] Mohammad H. Shekarriz, Dhananjay Thiruvady, Asef Nazari, and Rhyd Lewis. Soft happy colourings and community structure of networks. *Computers & Operations Research*, 174:106893, 2025.

[2] Mohammad H. Shekarriz, Dhananjay Thiruvady, Asef Nazari, and Wilfried Imrich. Local search improvements for soft happy colouring. *Preprint available on ArXiv: 2506.19284*, 2025.

[3] Peng Zhang and Angsheng Li. Algorithmic aspects of homophyly of networks. *Theoretical Computer Science*, 593:117–131, 2015.

List strong edge-colouring

B. Lužar(1), E. Máčajová(2), R. Soták(3), D. Švecová(3)

(1) Faculty of Information Studies, Novo mesto, Slovenia.
(2) Comenius University, Bratislava, Slovakia.
(3) Pavol Jozef Šafárik University, Košice, Slovakia.

A strong edge-coloring of a graph is an edge-coloring in which every color class is an induced matching. The least number of colors needed for a strong edge-coloring of a graph is the *strong chromatic index*.

We consider the list version of the coloring and prove that the list strong chromatic index of graphs with maximum degree 3 is at most 10. This bound is tight and improves the previous bound of 11 colors ([1]).

Next, we consider graphs with maximum degree 4, where the best known bound for the list strong edge-coloring is 22 ([2]). We improve this result and establish an upper bound of 21 for the strong list chromatic index of subquartic graphs. Since there exist subquartic graphs whose strong chromatic index is 20, our bound is only one above the best possible.

References

[1] T. Dai, G. Wang, D. Yang, and G. Yu, Strong list-chromatic index of subcubic graphs, *Discrete Math.*, 341(12) (2018), 3434–3440.

[2] B. Zhang,Y. Chang, J. Hu, M. Ma, D. Yang, List strong edge-coloring of graphs with maximum degree 4. *Discrete Math.* 343(6), 111854 (2020).

Irreducibility in distinguishing colourings

M. Stawiski[(1)]

[(1)] AGH University, Kraków, Poland

We investigate the role of the Axiom of Choice and its weaker forms in distinguishing and proper colourings. In particular, we formulate conditions equivalent to AC in terms of such colourings in both vertex and edge variants. Moreover, we study the notion of *irreducible* distinguishing colourings, i.e. distinguishing colourings such that no two non-empty classes of colours may be merged to obtain another distinguishing colourings. One may view such as distinguishing colourings for which no colour is abundant.

Normal edge coloring

I. Fabrici[1], B. Lužar[2], E. Máčajová[3], R. Soták[1], D. Švecová[1]

[1] P.J. Šafárik University, Košice, Slovensko
[2] Faculty of Information Studies, Novo mesto, Slovenia
[3] Comenius University, Bratislava, Slovakia

A normal edge coloring of a cubic graph is a proper edge coloring, in which every edge is adjacent to edges colored with four distinct colors or to edges colored with two distinct colors. It is conjectured that 5 colors suffices for a normal edge coloring of any bridgeless cubic graph and this statement is equivalent to the Petersen Coloring Conjecture. Currently, we only know that any cubic graph admits a normal edge coloring with at most 7 colors.

We present new results regarding the normal coloring of special graph classes. In the second part, we introduce the study of the list version of the normal edge coloring. It turns out to be more restrictive and consequently more colors are needed. In particular, we show that there are cubic graphs which need at least 9 colors for a list normal edge coloring and there are bridgeless cubic graphs which need at least 8 colors.

Quasi-majority neighbor sum distinguishing edge-colorings

R. Kalinowski(1), M. Pilśniak(1), E. Sidorowicz(2), E. Turowska(2)

(1) AGH University of Kraków, Kraków, Poland
(2) University of Zielona Góra, Zielona Góra, Poland

An edge-coloring c of a graph G defines in a natural way a vertex-coloring $\sigma_c : V(G) \to \mathbb{N}$ by $\sigma_c(v) = \sum_{u \in N_G(v)} c(vu)$ for each $v \in V(G)$. The edge-coloring c is called neighbor sum distinguishing if $\sigma_c(u) \neq \sigma_c(v)$ for every $uv \in E(G)$. This type of edge-coloring is related to the 1-2-3 Conjecture, proved by Keusch [1].

We study neighbor sum distinguishing edge-coloring under additional local constraint, requiring the edge-coloring to be quasi-majority. A k-edge-coloring of G is called quasi-majority if for every $v \in V(G)$ and every $\alpha \in [k]$, at most $\left\lceil \frac{d(v)}{2} \right\rceil$ edges incident to v are colored with α.

A k-edge-coloring of G is called quasi-majority neighbor sum distinguishing if it is quasi-majority and neighbor sum distinguishing. The smallest k for which G admits such a coloring is denoted by $\chi_{\sum}^{QM}(G)$. A graph is nice if it has no component isomorphic to K_2. We show that $\chi_{\sum}^{QM}(G) \leq 12$ for every nice G. This bound improves to 6 for nice bipartite graphs and to 7 for nice graphs of maximum degree at most four. Moreover, we determine the exact value of $\chi_{\sum}^{QM}(G)$ for complete graphs, complete bipartite graphs, and trees.

References

[1] R. Keusch, A Solution to the 1-2-3 Conjecture, *J. Combin. Theory Ser. B* 166 (2024) 182–202.

Odd independent sets and strong odd colorings of graphs

Y. Caro[(1)], M. Petruševski[(2)], R. Škrekovski[(3)], Zs. Tuza[(4,5)]

[(1)] University of Haifa-Oranim, Tivon, Israel
[(2)] Ss. Cyril and Methodius University in Skopje, Macedonia
[(3)] University of Ljubljana, Slovenia
[(4)] University of Pannonia, Veszprém, Hungary
[(5)] Alfréd Rényi Institute of Mathematics, Budapest, Hungary

We say that an $S \subset V(G)$ is an odd independent set in graph G if it is independent (induces no edges) and every vertex in $V \setminus S$ is adjacent either to no vertex of S or to an odd number of vertices of S. The largest cardinality of such a set is termed the odd independence number of G.

A strong odd coloring of G is a partition of the vertex set into odd independent sets; the corresponding parameter (minimum number of colors) is called strong odd chromatic number.

Beside many results concerning these notions, we also offer a large number of open problems for future research.

On high-girth high-chromatic subgraphs of Burling graphs

Seth Pettie[(1)], Gábor Tardos[(2)], Bartosz Walczak[(3)]

[(1)] University of Michigan, Ann Arbor, USA
[(2)] HUN-REN Rényi Institute, Budapest, Hungary
[(3)] Jagiellonian University, Kraków, Poland

A well-known conjecture of Erdős and Hajnal asserts that for any g and k, there is a finite number $f(g,k)$ such that every graph with chromatic number at least $f(g,k)$ contains a subgraph with girth at least g and chromatic number at least k. Rödl (1977) proved the conjecture for $g = 4$, in particular showing that $f(4,k)$ is bounded from above by a tower of ks of height $O(k^2 \log k)$. The conjecture remains open for $g \geq 5$.

A construction of triangle-free high-chromatic graphs due to Burling (1965) was used in the last decade to provide counterexamples to several conjectures and was shown to have various unexpected properties. We show that while Burling graphs do satisfy the aforesaid Erdős–Hajnal conjecture, they provide the first non-trivial lower bound on the growth of $f(g,k)$. Specifically, if $T(k)$ denotes the tower of 2s of height k, we prove that the Burling graph with chromatic number $T(k - O(1))$ has no subgraph with girth 5 and chromatic number at least k, showing that $f(5,k) > T(k - O(1))$.

A key tool for the proof is a combinatorial game in which two players, Builder and Chooser, alternate turns to build a graph vertex by vertex as follows: Builder introduces a new vertex with edges to all previous vertices and then partitions the entire edge set into two subsets, after which Chooser deletes one of the two subsets. Builder attempts to build a clique of size k, while Chooser attempts to prevent that. We prove upper and lower bounds of the form $T(k \pm O(1))$ on how many turns Builder needs to guarantee a k-clique.

Some recent results on modular product

Iztok Peterin[(1,2)]

(1) Faculty of Electical Engineering and Computer Science, University of Maribor
(2) Institute of Mathematics, Physics and Mechanics, Ljubljana

A product of two graphs G and H has a vertex set $V(G) \times V(H)$. For edges, we consider three objects in every factor for the definition of an edge in a product: either a vertex or an edge or a non-edge. Combining the mention objects from one factor with the same objects in the other factor gives eight different possibilities (notice that vertex by vertex must be ignored to avoid loops) to have an edge or a non-edge in a product. So, all together $2^8 = 256$ different graph products exists and four of them—Cartesian $G \Box H$, strong $G \boxtimes H$, direct $G \times H$ and lexicographic $G \circ H$—gain a special status and are called standard products.

Among all graph products only 10 are associative and commutative. We join them into pairs of a product $G * H$ together with its complementary product $G \overline{*} H \overset{\text{def}}{=} \overline{\overline{G} * \overline{H}}$ where $\overline{G}$ is the complement of a graph G. Six of them represent either a standard product (Cartesian, strong or direct) or its complementary product, next two are empty and complete product and finally modular product $G \diamond H$, where $E(G \diamond H) = E(G \Box H) \cup E(G \times H) \cup E(\overline{G} \times \overline{H})$, and its complementary product.

We present several recent results on distance [2], domination number [1] and independence sets of modular products with respect to some properties of their factors.

References

[1] S. Bermudo, I. Peterin, J. Sedlar, R. Škrekovski, Domination number of modular product graphs, Comput. Applied Math. 44(1) (2025) #65.

[2] C.X. Kang, A. Kelenc, I. Peterin, E.Yi, On Distance and Strong Metric Dimension of the Modular Product, Bulletin Malaysian Math. Sciences Soc. 48(1) (2025) #20.

Integrity of grids

J. Kozik[(1)], A. Żak[(2)]

[(1)] AGH University, Krakow, Poland
[(2)] AGH University, Krakow, Poland

The integrity of a graph $G = (V, E)$ is defined as the smallest sum $|S| + m(G - S)$, where S is a subset of the set V, and $m(H)$ denotes the order of the largest component of the graph H.

Benko, Ernst, and Lanphier provided and proved an asymptotic bounds for planar graphs in terms of the order of the graph. We prove asymptotic results concerning two-dimensional grid-graphs.

References

[1] D. Benko, C. Ernst, D. Lanphier, Asymptotic bounds on the integrity of graphs and separator theorems for graphs, *SIAM Journal on Discrete Mathematics* 23 2009 pp.265–277.

[2] A. Żak, A note of integrity, *Discrete Applied Mathematics* 341 2023, pp.55–59.

Construction of k-matchings in graph products

Anna Lindeberg[1], Marc Hellmuth[1]

[1] Dept. of Mathematics, Stockholm University, Sweden

The study of classical graph invariants—such as chromatic, domination, and independence numbers—in graph products has received significant attention. Here, we focus on variations of matchings in the four standard graph products: Cartesian, strong, direct and lexicographic. Specifically, we define a subset $M \subseteq E$ of a graph $G = (V, E)$ as a *k-matching* if the edges in M induce a k-regular subgraph of G.

Summarizing results in [1], we present explicit constructions of k-matchings in graph products $G \star H$, utilizing k_G-matchings M_G and k_H-matchings M_H from the factor graphs G and H. Although these constructions do not always yield maximum k-matchings for the product, they achieve the largest possible size among all k-matchings that are weak-homomorphism preserving – meaning that matched edges in the product never project onto unmatched edges in the factors.

References

[1] A. Lindeberg, M. Hellmuth. Construction of k-matchings in graph products. *Art Discrete Appl. Math.* 6(2) (2022). DOI:10.26493/2590-9770.1462.b03.

Planar and polyhedral graphs as Kronecker and Sierpiński products

R. Maffucci(1), R. De March(2)

(1) Università di Torino, Italy
(2) Data scientist, Torino, Italy

We will discuss graph products that are planar/polyhedral. The first part of the talk focuses on the Kronecker (direct, tensor) product [2], Figure 1. We also consider simultaneous products [3].

The second part of the talk [4] is on the Sierpiński product, recently introduced in [1].

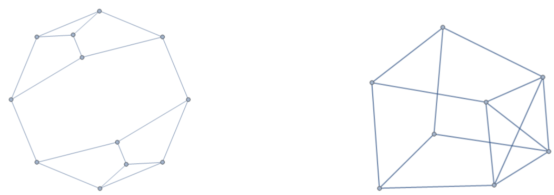

Figure 1: Illustrations of a planar (left) and a non-planar (right) factor for planar, 3-connected Kronecker products.

References

[1] J.Kovic, T.Pisanski, S.S.Zemljic, A.Zitnik. 'The Sierpiński product of graphs'. *Ars Math. Contemp.* 2023, 23(1).

[2] R.Maffucci, 'Classification and Construction of Planar, 3-Connected Kronecker Products', *arXiv:2402.01407*.

[3] R.De March, R.Maffucci, 'Cancellation and regularity for planar, 3-connected Kronecker products' *arXiv:2411.13473*.

[4] R.Maffucci, 'Regularity and separation for Sierpiński products of graphs', *arXiv:2506.16864*.

Group distance magic cubic graphs

S. Cichacz[(1)], Š. Miklavič[(2,3)]

[(1)] AGH University of Krakow, Poland
[(2)] Andrej Marušič Institute, University of Primorska, Koper, Slovenia
[(3)] Institute od mathematics, physics and mechanics, Ljubljana, Slovenia

A Γ-*distance magic labeling* of a graph $G = (V, E)$ with $|V| = n$ is a bijection ℓ from V to an Abelian group Γ of order n, for which there exists $\mu \in \Gamma$, such that the weight $w(x) = \sum_{y \in N(x)} \ell(y)$ of every vertex $x \in V$ is equal to μ. In this case, the element μ is called the *magic constant of* G. A graph G is called a *group distance magic* if there exists a Γ-distance magic labeling of G for every Abelian group Γ of order n.

In this talk, we focus on cubic Γ-distance magic graphs as well as some properties of such graphs.

This work was partially supported by program "Excellence initiative – research university" for the AGH University.

Self-reverse distance magic labeling

P. Kovář[(1)], K. Rozman[(2,3)], P. Šparl[(2,4,5)]

[(1)] Department of Applied Mathematics, VSB, Ostrava, Czech Republic
[(2)] IMFM, Ljubljana, Slovenia
[(3)] UP FAMNIT, Koper, Slovenija
[(4)] UL PEF, Ljubljana, Slovenia
[(5)] UP IAM, Koper, Slovenija

According to a nonstandard definition introduced in 2021, a distance magic labeling ℓ of a regular graph of order n is a bijection from its vertex set to the set of integers of the arithmetic progression from $1-n$ to $n-1$ with common difference 2, such that the sum of the labels of the neighbors of each vertex is zero. Such a labeling is called *self-reverse* if, for any pair of vertices u and v, u is adjacent to v if and only if the vertices with labels $-\ell(u)$ and $-\ell(v)$ are adjacent.

In this talk, we present the motivation for studying self-reverse distance magic labelings. We focus on self-reverse distance magic labelings in the case of tetravalent graphs providing several examples and a complete classification of all orders for which a tetravalent graph admitting such a labeling exists. The classification is obtained via a novel construction that produces a (tetravalent) distance magic graph from two given (tetravalent) distance magic graphs. We also discuss the existence of graphs admitting a self-reverse distance magic labeling among some well-known families of tetravalent graphs.

Pushing for Irregularity

D. Rautenbach(1), L. Schwartze(1), F. Werner(1)

(1) Ulm University, Ulm, Germany

Let G be a simple graph with maximum degree $\Delta(G)$ and no component of order 2. Bensmail, Marcille, and Orenga [1] introduce the notion of a *pushing scheme* $\rho : V(G) \to \mathbb{N}_0$ with induced edge labeling

$$\ell : E(G) \to \mathbb{N}, \quad uv \mapsto 1 + \rho(u) + \rho(v).$$

ρ should be chosen such that the induced vertex labeling

$$\sigma : V(G) \to \mathbb{N}_0, \quad v \mapsto \sum_{u \in N_G(v)} \ell(uv)$$

is a proper vertex coloring. Bensmail et al. conjecture that such a
$\rho : V(G) \to \{0, 1, \ldots, \Delta(G)\}$ exists for every graph G. We prove their conjecture for a few graph classes [2].

References

[1] J. Bensmail, C. Marcille, and M. Orenga, Pushing Vertices to Make Graphs Irregular, hal-04810955.

[2] D. Rautenbach, L. Schwartze, F. Werner, Coloring by Pushing Vertices, 2025, `https://arxiv.org/abs/2505.05252`.

On rainbow caterpillars

S. Cichacz(1), B. Stołowska(1), M. Woźniak(1)

(1) AGH, Krakow, Poland

Given a finite Abelian group $(A, +)$, consider a tree T with $|A|$ vertices. The labeling $f\colon V(T) \to A$ of the vertices of some graph G induces an edge labeling in G, thus the edge uv receives the label $f(u) + f(v)$. The tree T is A-rainbow colored if f is a bijection and edges have different colors. In this paper, we give necessary and sufficient conditions for a caterpillar with three spine vertices to be A-rainbow, when A is an elementary p-group.

References

[1] C. Delorme, *Cayley digraphs and graphs*, European Journal of Combinatorics 34(8) (2013) 1307–1315.

[2] M. Hovey, *A-cordial graphs*, Discrete Math., 93 (1991) 183–194.

[3] R.E. Jamison, W.B. Kinnersley, *Rainbow spanning trees in abelian groups*, Journal of Algebraic Combinatorics 56 (2022) 5–21.

[4] R. Patrias, O. Pechenik, *Path-cordial abelian groups*, Austral. J. Comb. 80(1) (2021) 157–166.

Zombies on the Grid

Omer Angel[1], Paweł Prałat[2], Peter Winkler[3]

[1] University of British Columbia, Vancouver, BC, Canada
[2] Toronto Metropolitan University, Toronto, ON, Canada
[3] Dartmouth, Hanover, NH, USA

In Zombies and Survivors, a set of zombies attempts to eat a lone survivor, Mindy, loose on a given connected graph. The zombies randomly choose their initial location, and during the course of the game, move directly toward Mindy. At each round, they move to the neighbouring vertex that minimizes the distance to Mindy; if there is more than one such vertex, then they choose one uniformly at random. Mindy attempts to escape from the zombies by moving to a neighbouring vertex or staying on her current vertex. The zombies win if eventually one of them eats Mindy by landing on her vertex; otherwise, Mindy wins. The zombie number $z(G)$ of a graph G is the minimum number of zombies needed to play such that the probability that they win is at least $1/2$.

In this paper, we investigate the game played on toroidal grids $C_n \square C_n$. In particular, we show that asymptotically almost surely $z(C_n \square C_n) = \Omega(n/\log^c n)$ for some constant c and that $z(C_n \square C_n) = O(n^{3/2})$.

Algebraic relations for permutons

O. Angel[1], F. Skerman[2], P. Winkler[3]

[1] UBC, Vancouver, Canada
[2] Uppsala niversity, Uppsala, Sweden
[3] Dartmouth, Hanover, US

Permutons have emerged as a highly successful method for studying structure and certain properties of large permutations. In particular they are closely related to pattern densities. However, they are lacking when considering algebraic properties of the permutations such as cycle structure, order, and permutation compositions. We investigate what can still be said about products of permutations in the permuton limit.

On random regular graphs and the Kim-Vu Sandwich Conjecture

Natalie Behague(1), Daniel Il'kovič(2), Richard Montgomery(1)

(1) University of Warwick, Coventry, UK

(2) Leipzig University, Leipzig, Germany

The random regular graph $G_d(n)$ is selected uniformly at random from all d-regular graphs on n vertices. This model is a lot harder to study than the Erdős-Renyi binomial random graph model $G(n, p)$ as the probabilities of edges being present are not independent. However, in the regime $d \gg \log n$, various graph properties including Hamiltonicity and chromatic number were shown (with hard work) to be the same in $G_d(n)$ as in $G(n, p)$ with $np = d$. This inspired Kim and Vu [2] to conjecture that when $d \gg \log n$ it is possible to 'sandwich' the random regular graph $G_d(n)$ between two Erdős-Renyi random graphs with similar edge density. A proof of the conjecture would immediately imply many results about monotone graph properties of $G_d(n)$ in this dense regime, and would unify all the previous separate hard-won results.

Various authors have proved weaker versions of this conjecture with incrementally improved bounds on d. The previous state of the art was due to Gao, Isaev and McKay [1] who proved the conjecture for $d \gg \log^4 n/(\log \log n)^3$. I will talk about our new improvement of this result.

References

[1] P. Gao, M. Isaev, and B. McKay. Kim-Vu's sandwich conjecture is true for $d \geq \log^4 n$. *arXiv preprint arXiv:2011.09449*, 2020.

[2] J. H. Kim, V. H. Vu, Sandwiching random graphs: universality between random graph models, *Advances in Mathematics* 188(2):444–469, 2004.

Packing edge-disjoint copies of a fixed graph in the random geometric graph

P. Bennett[(1)], R. Cushman[(2)], A. Dudek[(1)], X. Pérez-Giménez[(3)]

[(1)] Western Michigan University, Kalamazoo, USA
[(2)] University of Tennessee, Knoxville, Knoxville, USA
[(3)] University of Nebraska–Lincoln, Lincoln, USA

Fix a graph H. Can we cover almost all the edges of a d-dimensional random geometric graph with edge-disjoint copies of H? Perhaps surprisingly, we will see that for some choices of H, d the answer is "no," even if the random geometric graph is dense. We also show that the answer is "yes" for many choices of H, d.

Connectivity thresholds for superpositions of Bernoulli random graphs

M. Bloznelis[(1)]

[(1)] Vilnius University, Vilnius, Lithuania

Let $G_1, \ldots, G_m$ be independent identically distributed Bernoulli random subgraphs of the complete graph $\mathcal{K}_n$. For $k = 1, 2, \ldots,$ we show the k-connectivity threshold as $n, m \to +\infty$ for the union graph $\cup_{i=1}^m G_i$ defined on the vertex set of $\mathcal{K}_n$. For $k = 2, 3, \ldots$ we observe two different threshold behaviors: one for the unions of cliques and the other one for the (remaining) case where each G_i has a vertex of degree 1 with positive probability. Results for the case $k = 1$ have been reported in [1, 2].

References

[1] D.Ardickas, M.Bloznelis, Connectivity threshold for superpositions of Bernoulli random graphs, *Discrete Math.* 2025, 348, 114684.

[2] M.Bloznelis, D. Marma. R. Vaicekauskas, Connectivity threshold for superpositions of Bernoulli random graphs.II, *Acta Math. Hung.* 2025, 175, 352 - 375.

Loose paths in random ordered hypergraphs

Andrzej Dudek[1]

[1] Western Michigan University

Consider the random r-uniform hypergraph $H = H^{(r)}(n,p)$. An *ordered loose path* is a sequence of edges $E_1, E_2, \ldots, E_\ell$ of H such that $\max\{j \in E_i\} = \min\{j \in E_{i+1}\}$ for $1 \leq i < \ell$. In this talk we establish fairly tight bounds on the length of the longest ordered loose path in H that hold with high probability.

This is a joint work with Alan Frieze and Wesley Pegden.

Sharp Thresholds Imply Circuit Lower Bounds

D. Gamarnik[(1)]

[(1)] MIT, Cambridge, US

We show that sharp thresholds for Boolean functions imply circuit lower bounds. More formally we show that any Boolean function exhibiting a sharp enough threshold at an arbitrary threshold cannot be computed by Boolean circuits of bounded depth and polynomial size. This verifies a conjecture put forward earlier in the survey by Kalai and Safra. Our result is of particular interest in the sparse random graph setting where the main tool for bounding circuit depth, namely Linial-Mansour-Nisan (LMN) theorem does not apply. We redeem the power of LMN theorem by creating simple dense-to-sparse circuit gadgets. Our result will be illustrated using two models: independent sets in sparse random graphs and random 2-SAT model.

Joint work with Elchanan Mossel (MIT) and Ilias Zadik (Yale University).

References

[1] D.Gamarnik, E.Mossel, I. Zadik, Sharp Thresholds Imply Circuit Lower Bounds: from random 2-SAT to Planted Clique, *Israel Journal of Mathematics, to appear.* 2007 pp.145-161.

Preferential attachment trees with vertex death

Markus Heydenreich[(1)], Bas Lodewijks[(2)]

[(1)] University of Augsburg, Augsburg, Germany
[(2)] University of Sheffield, Sheffield, United Kingdom

Preferential attachment models are a popular class of random graphs that have received a wealth of attention in the last decades and are often used to model evolving networks. In such models, new vertices are added to the graph sequentially and new vertices are more likely to make connections with existing vertices that have a large degree.

In recent work, we study a general preferential attachment model where vertices can both be added but can also be 'killed'. Such killed vertices can no longer make new connections, whereas 'alive' vertices continue to make new connections. This models evolving networks that can both increase as well as decrease in size.

We focus on *persistence of the maximum degree*: are the oldest alive vertices also the ones with largest degree? We uncover a novel regime in which killing of vertices makes such persistence entirely impossible.

Hitting times and the power of choice for random geometric graphs

D. Ignasiak[(1)], L. Lichev[(1)]

[(1)] Technical University of Vienna, Vienna, Austria

We consider a random geometric graph process where random points $(X_i)_{i\geq 1}$ are embedded consecutively in the d-dimensional unit torus $\mathbb{T}^d$, and every two points at distance at most r form an edge. As $r \to 0$, we confirm that well-known hitting time results for k-connectivity (with $k \geq 1$ fixed) and Hamiltonicity in the Erdős-Rényi graph process also hold for the considered geometric analogue. Moreover, we exhibit a sort of probabilistic monotonicity for each of these properties.

We also study a geometric analogue of the power of choice where, at each step, an agent is given two random points sampled independently and uniformly from $\mathbb{T}^d$ and has to add exactly one of them to the already constructed point set. When the agent is allowed to make their choice with the knowledge of the entire sequence of random points (offline 2-choice), we show that they can construct a connected graph at the first time t when none of the first t pairs of proposed points contains two isolated vertices in the graph induced by $(X_i)_{i=1}^{2t}$, and maintain connectivity thereafter. We also derive analogous results for k-connectivity and Hamiltonicity. This shows that each of the said properties can be attained two times faster (time-wise) and with four times fewer points in the offline 2-choice process compared to the 1-choice process.

In the online version where the agent only knows the process until the current time step, we show that k-connectivity and Hamiltonicity cannot be significantly accelerated (time-wise) but may be realised on two times fewer points compared to the 1-choice analogue.

Recovery of spatial vertex features in noisy SPA model graphs

Jordan Barrett(1), Jeannette Janssen(1), Aaron Smith(2)

(1) Dalhousie University, Halifax, Canada
(2) University of Ottawa, Ottawa, Canada

The graph matching problem is that of identifying vertices in two graphs that are independent perturbations of a single random graph. Inspired by an approach recently introduced in a paper by Liu and Austern [1], we consider graphs generated via a geometric random graph model. In particular, features associated with each vertex can be interpreted as giving the spatial position of the vertex, and the formation of the graph is informed by the positions of the vertices. Noisy versions of the vertex features are assumed to be given; an important step in graph matching is then that of estimating the true positions of the vertices. We apply this approach to the spatial preferential attachment (SPA) model [2], which generates sparse spatial graphs with a power law degree distribution. We propose a two-step approach to estimating the spatial positions of the vertices, and derive bounds on the noise parameters under which our method is an efficient estimator for the positions.

References

[1] S. Liu, M. Austern, Perfect Recovery for Random Geometric Graph Matching with Shallow Graph Neural Networks, *arXiv:2402.07340* (2025).

[2] W. Aiello, A. Bonato, C. Cooper, J.Janssen, P. Prałat , A spatial web graph model with local influence regions, *Internet Mathematics* 5 (2009), pp. 175–196.

On the threshold for random triangulations inside large convex polygons

Brett Kolesnik[1]

[1] University of Warwick, Coventry, United Kingdom

Start with a convex polygon and then add a random graph of edges inside. We will discuss some results concerning the critical point at which a triangulation of the polygon emerges. Joint work with Georgii Zakharov (Oxford) and Maksim Zhukovskii (Sheffield).

Monochromatic matchings in almost-complete and random hypergraphs

Lior Gishboliner(1), Stefan Glock(2), Peleg Michaeli(3), Amedeo Sgueglia(2)

(1) University ot Toronto, Canada
(2) University of Passau, Germany
(3) University of Oxford, UK

The Ramsey number for matchings in graphs was determined by Cockayne and Lorimer [1] and later extended to hypergraphs by Alon, Frankl, and Lovász [2], who showed that every q-colouring of the complete r-uniform hypergraph on n vertices contains a monochromatic matching of size $\lfloor((n+q-1)/(r+q-1))\rfloor$.

In this talk I will present two extensions of this classical result. The first is a **defect version**, which asserts that every q-colouring of an *almost-complete* uniform hypergraph contains a monochromatic matching of comparable size to that in the complete case. The second is a **transference principle**, which demonstrates that a monochromatic matching of comparable size exists, with high probability, in any q-colouring of a *sparse, random* uniform hypergraph with sufficiently large constant average degree.

The proofs combine methods from extremal set theory with a variant of the weak hypergraph regularity lemma.

References

[1] E. J. Cockayne, P. J. Lorimer, The Ramsey number for stripes, *J. Austral. Math. Soc.* **19** (1975), 252–256.

[2] N. Alon, P. Frankl, L. Lovász, The chromatic number of Kneser hypergraphs, *Trans. Amer. Math. Soc.* **298** (1986), 359–370.

Homogeneous substructures in random ordered uniform matchings

Andrzej Ruciński[(1)]

[(1)] Adam Mickiewicz University, Poznań, Poland

An ordered r-uniform matching of size n is a collection of n pairwise disjoint r-subsets (edges) of a linearly ordered set of rn vertices. For $n = 2$, such a matching is called an *r-pattern*, as it represents one of $\frac{1}{2}\binom{2r}{r}$ ways two disjoint edges may intertwine. Given a set $\mathcal{P}$ of r-patterns, a *$\mathcal{P}$-clique* is a matching with all pairs of edges belonging to $\mathcal{P}$.

I will present recent results determining asymptotically the largest size of a $\mathcal{P}$-clique in a *random* ordered r-uniform matching, for several classes of sets of patterns $\mathcal{P}$. This is joint work with A. Dudek, J. Grytczuk, and J. Przybyło.

3-colourability, diamonds and butterflies

Nadzieja Hodur[(1)], Monika Pilśniak[(1)], Magdalena Prorok[(1)],
Ingo Schiermeyer[(1,2)]

[(1)] AGH University of Krakow, Kraków, Poland
[(2)] TU Bergakademie Freiberg, Freiberg, Germany

The 3-colourability problem is an NP-complete problem which remains NP-complete for graphs with maximum degree four, for claw-free graphs, and even for (claw,diamond)-free graphs. In this talk we will consider induced subgraphs, among them are the *claw* ($K_{1,3}$), the *diamond* (the graph $K_4 - e$), the *butterfly* (two triangles sharing a vertex), and the generalized *net* $N_{i,j,k}$ (a triangle with three attached paths with i, j, k edges).

Our main result is a complete characterization of all 3-colourable $(claw, diamond, H)$-free graphs for $H \in \{N_{1,1,1}, N_{1,1,2}, N_{1,2,2}, N_{2,2,2}\}$. We will present a description of all non 3-colourable $(claw, diamond, H)$-free graphs for $H \in \{N_{1,1,1}, N_{1,1,2}, N_{1,2,2}, N_{2,2,2}\}$ in terms of butterflies. Moreover, we will show extensions of this characterization to larger graph classes.

References

[1] B. Randerath and I. Schiermeyer, *Vertex colouring and forbidden subgraphs - a survey*, **Graphs and Combinatorics** 20 (2004) 1-40.

[2] I. Schiermeyer and B. Randerath, *Polynomial χ-Binding Functions and Forbidden Induced Subgraphs: A Survey*, **Graphs and Combinatorics** 35 (1) (2019) 1–31.

On Mycielskians of digraphs

M. Borowiecka-Olszewska(1), E. Drgas-Burchardt(1),
N.Y. Javier Nol(2), R. Zuazua(2)

(1) University of Zielona Góra, Poland
(2) National Autonomous University of Mexico, Mexico

Let D be a digraph on vertices $v_1, \ldots, v_n$. The *Mycielskian* of D, denoted $M(D)$, is obtained from D by adding an independent set of vertices $V' = \{v'_1, \ldots, v'_n\}$ and one extra vertex x. For every arc (v_i, v_j) of D, the arcs (v_i, v'_j) and (v'_i, v_j) are added, and finally arcs from x to all vertices of V' are included. In a natural way, a sequence of digraphs $\{M_p(D)\}_{p\geq 0}$ is defined by $M_0(D) = D$, $M_1(D) = M(D)$, and $M_p(D) = M(M_{p-1}(D))$ for $p \geq 2$, where $M_p(D)$ is called the *p-th Mycielskian* of D.

For a digraph D, a set S of arcs is a *feedback arc set* if $D - S$ is acyclic, and the minimum size of such a set is denoted by $\tau_1(D)$. In this talk we focus on the parameter $\tau_1(M_p(D))$, as well as on the maximum number of arc-disjoint directed cycles in $M_p(D)$, denoted by $\nu_1(M_p(D))$, which is closely related to $\tau_1(M_p(D))$.

References

[1] B. Csonka, G. Simonyi, Shannon capacity, Lovász theta number and the Mycielski construction, *IEEE Trans. Inform. Theory* 2024 pp. 7632–7646.

Further progress on Wojda's conjecture

M. Ciśiński(1), A. Żak(1)

(1) AGH University, Cracow, Poland

Two digraphs of order n are said to pack if they can be found as edge-disjoint subgraphs of the complete digraph of order n. It is well established that if the sum of the sizes of the two digraphs is at most $2n-2$, then they pack, with this bound being sharp. However, it is sufficient for the size of the smaller digraph to be only slightly below n for the sum of their sizes to significantly exceed this threshold while still guaranteeing the existence of a packing.

In 1985, Wojda conjectured that for any $2 \leq m \leq n/2$, if one digraph has size at most $n-m$ and the other has size less than $2n - \lfloor n/m \rfloor$, then the two digraphs pack. It was previously known that this conjecture holds for $m = \Omega(\sqrt{n})$. We confirm it for $m \geq 26$.

References

[1] A. Benhocine, H.J. Veldman, A.P. Wojda, Packing of digraphs, Ars Combin. **22** (1986), 43–49.

[2] J. Konarski, A. Żak, Toward Wojda's conjecture on digraph packing, Opuscula Math. **37** no. 4 (2017), 589–595.

[3] A.P. Wojda, Research problems (Problem 69), Discrete Math. **57** (1985), 209–210.

[4] A.P. Wojda, M. Woźniak, Packing and extremal digraphs, Ars Combin. **20** B (1985), 71–73.

On coarse tree decompositions and coarse balanced separators

Tara Abrishami[(1)], Jadwiga Czyżewska[(2)], Kacper Kluk[(2)],
Marcin Pilipczuk[(2)], Michał Pilipczuk[(2)], Paweł Rzążewski[(2,3)]

[(1)] University of Hamburg, Germany
[(2)] University of Warsaw, Poland
[(3)] Warsaw University of Technology, Poland

It is known that there is a linear dependence between the treewidth of a graph and its *balanced separator number*: the smallest integer k such that for every weighing of the vertices, the graph admits a balanced separator of size at most k. We investigate whether this connection can be lifted to the setting of coarse graph theory, where both the bags of the considered tree decompositions and the considered separators should be coverable by a bounded number of bounded-radius balls.

As the first result, we prove that if an n-vertex graph G admits balanced separators coverable by k balls of radius r, then G also admits tree decompositions $\mathcal{T}_1$ and $\mathcal{T}_2$ such that:

- in $\mathcal{T}_1$, every bag can be covered by $\mathcal{O}(k \log n)$ balls of radius r; and
- in $\mathcal{T}_2$, every bag can be covered by $\mathcal{O}(k^2 \log k)$ balls of radius $r(\log k + \log\log n + \mathcal{O}(1))$.

As the second result, we show that if we additionally assume that G has doubling dimension at most m, then the functional equivalence between the existence of small balanced separators and of tree decompositions of small width can be fully lifted to the coarse setting.

Constructing algebraic expressions for lattice-structured digraphs

M. Korenblit[(1)]

[(1)] Holon Institute of Technology, Holon, Israel

We investigate relationship between algebraic expressions and *labeled two-terminal directed acyclic graphs* (labeled *st-dags*), in which each edge carries a unique label. An algebraic expression is referred to as an *st-dag expression* if it is algebraically equivalent to the sum of edge-label products over all spanning paths of the st-dag. The st-dags considered here are based on lattice structure and have $m \times n$ vertices (m rows and n columns). Examples are shown in Figure 2: (a) *grid graph* $G_{m,n}$, (b) *triangular grid* $T_{m,n}$, (c) *king graph* $K_{m,n}$. We treat m as a constant representing the graph's *depth*, while n determines its *size*. Our objective is to simplify the expressions for these graphs. To this end, we apply *backtracking* and *decomposition* methods (*BM* and *DM*, respectively) which generate expressions for these graphs, and we estimate the lengths of the generated expressions as functions of n. BM produces expressions of length $O(n^m)$ for both $G_{m,n}$ and $T_{m,n}$, and of exponential length in n for $K_{m,n}$. In contrast, DM yields more compact expressions: length $O\left(n \log^{m-1} n\right)$ for both $G_{m,n}$ and $T_{m,n}$ and length $O\left(n^{\log_2(4m-2)}\right)$ for $K_{m,n}$.

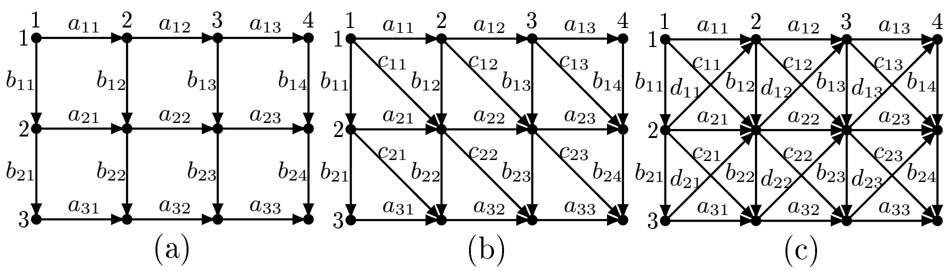

Figure 2: Lattice-structured digraphs.

A generalization of an ear decomposition and k-trees in highly connected star-free graphs

Shun-ichi Maezawa[(1)], Kenta Ozeki[(2)], Masaki Yamamoto[(3)], Takamasa Yashima[(4)]

[(1)] Nihon University, Tokyo, Japan
[(2)] Yokohama National University, Yokohama, Japan
[(3)] Seikei University, Tokyo, Japan
[(4)] Kanazawa Institute of Technology, Ishikawa, Japan

In this talk, we introduce a generalized version of an ear decomposition, called a j-spider decomposition, for j-connected star-free graphs with $j \geq 2$. Its application enables us to improve a previousely known sufficient condition for the existence of a k-tree in highly connected star-free graphs, where a k-tree is a spanning tree in which every vertex is of degree at most k. More precisely, we show that every j-connected $K_{1,j(k-2)+2}$-free graph has a k-tree for $k \geq j$, thereby improving a classical result of Jackson and Wormald [1] for $k \geq j$. Our approach differs from previous studies based on toughness-type arguments and instead relies on both a j-spider decomposition and a factor theorem related to Hall's marriage theorem.

This talk is based on the paper [2].

References

[1] B.Jackson and N.C.Wormald, k-walks of graphs, *Australas. J. Combin.* **2** (1990) pp. 135-146.

[2] S.Maezawa, K.Ozeki, M.Yamamoto, and T.Yashima, A generalization of an ear decomposition and k-trees in highly connected star-free graphs, arXiv:2508.05962.

H-kernels in 3-quasi-transitive digraphs

Carlos Cedillo-Ponce, Rocío Sánchez-López[1]

[1] Faculty of Sciences, National Autonomous University of Mexico, CDMX, México.

Let H be a digraph possibly with loops and D a digraph without loops whose arcs are colored with the vertices of H (D is said to be an H-colored digraph). A directed path P in D is said to be an H-path if and only if the consecutive colors encountered on P form a directed walk in H. An H-kernel of an H-colored digraph D is a subset of vertices of D, say N, such that for every pair of different vertices in N there is no H-path between them, and for every vertex u in $V(D) \setminus N$ there exists an H-path in D from u to N. D is said to be 3-quasi-transitive if for every pair of vertices u and v of D, the existence of a directed path of length 3 from u to v implies that $\{(u,v),(v,u)\} \cap A(D) \neq \emptyset$. In this talk we show a result regarding the existence of H-kernels in 3-quasi-transitive digraphs; mainly the existence of H-kernels is guaranteed by means of sufficient conditions on the directed cycles of length 3 and 4.

Representing Distance-Hereditary Graphs with Trees

Guillaume E. Scholz[(1)]

[(1)] Free agent, Leipzig, Germany

Cographs are precisely the undirected graphs that can be represented by a vertex-labelled tree, that is, a pair (T, t), where T is a rooted tree and t is a labelling of the vertices of T into the set $\{0, 1\}$. Specifically, we say that the pair (T, t) explains G if T has leaf set $V(G)$ and, for any two distinct vertices x and y of G, x and y are joined by an edge in G if and only if the least common ancestor of x and y in T has label 1 via t.

Recently [1], the class of arboreal networks was introduced as a generalization of rooted trees. Arboreal networks are directed, acyclic graphs whose underlying, undirected graph is a tree. Intuitively, they are rooted trees which can have more than one root. This led to the question of characterizing those undirected graphs G that can be explained by a vertex-labelled arboreal network (N, t), in the same way cographs are explained by vertex-labelled trees. Interestingly, this is a well known and well studied class of graphs [2].

References

[1] K. T. Huber, V. Moulton and G. E. Scholz, Shared ancestry graphs and symbolic arboreal maps, *SIAM Journal on Discrete Mathematics*, 2024, 38(4): 2553-2577.

[2] G. E. Scholz, Representing distance-hereditary graphs with multi-rooted trees, *Graphs and Combinatorics* 2025 (in press).