Quasi-majority neighbor sum distinguishing edge-colorings

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An edge-coloring c of a graph G defines in a natural way a vertex-coloring $\sigma_c : V(G) \to \mathbb{N}$ by $\sigma_c(v) = \sum_{u \in N_G(v)} c(vu)$ for each $v \in V(G)$. The edge-coloring c is called neighbor sum distinguishing if $\sigma_c(u) \neq \sigma_c(v)$ for every $uv \in E(G)$. This type of edge-coloring is related to the 1-2-3 Conjecture, proved by Keusch [1].

We study neighbor sum distinguishing edge-coloring under additional local constraint, requiring the edge-coloring to be quasi-majority. A k-edge-coloring of G is called quasi-majority if for every $v \in V(G)$ and every $\alpha \in [k]$, at most $\left\lceil \frac{d(v)}{2} \right\rceil$ edges incident to v are colored with α.

A k-edge-coloring of G is called quasi-majority neighbor sum distinguishing if it is quasi-majority and neighbor sum distinguishing. The smallest k for which G admits such a coloring is denoted by $\chi_{\sum}^{QM}(G)$. A graph is nice if it has no component isomorphic to K_2. We show that $\chi_{\sum}^{QM}(G) \leq 12$ for every nice G. This bound improves to 6 for nice bipartite graphs and to 7 for nice graphs of maximum degree at most four. Moreover, we determine the exact value of $\chi_{\sum}^{QM}(G)$ for complete graphs, complete bipartite graphs, and trees.

References

[1] R. Keusch, A Solution to the 1-2-3 Conjecture, *J. Combin. Theory Ser. B* 166 (2024) 182–202.