Soft Happy Colouring

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For a coloured graph G and $0 \leq \rho \leq 1$, a vertex v is ρ-happy if at least $\rho \deg(v)$ of its neighbours share its colour. The soft happy colouring problem seeks a colouring σ that extends a given precolouring and maximises the number of ρ-happy vertices [3]. This NP-hard problem is closely linked to community detection in graphs. For example, for a graph in the stochastic block model (SBM) and for suitable ρ, with high probability, complete soft happy colourings can be achieved by the planted community structure [1]. Moreover, for $0 \leq \rho_1 < \rho_2 \leq 1$, complete ρ_2-happy colourings achieve higher detection accuracy than complete ρ_1-happy colourings, and when ρ surpasses a critical threshold, it is unlikely to find a complete ρ-happy colouring with near-equal class sizes [2]. Finally, we survey existing algorithms and propose novel heuristic, local search, evolutionary, metaheuristic, and matheuristic approaches that enhance solution quality for soft happy colouring.

References

[1] Mohammad H. Shekarriz, Dhananjay Thiruvady, Asef Nazari, and Rhyd Lewis. Soft happy colourings and community structure of networks. *Computers & Operations Research*, 174:106893, 2025.

[2] Mohammad H. Shekarriz, Dhananjay Thiruvady, Asef Nazari, and Wilfried Imrich. Local search improvements for soft happy colouring. *Preprint available on ArXiv: 2506.19284*, 2025.

[3] Peng Zhang and Angsheng Li. Algorithmic aspects of homophyly of networks. *Theoretical Computer Science*, 593:117–131, 2015.