Pushing for Irregularity

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Let G be a simple graph with maximum degree $\Delta(G)$ and no component of order 2. Bensmail, Marcille, and Orenga [1] introduce the notion of a *pushing scheme* $\rho : V(G) \to \mathbb{N}_0$ with induced edge labeling

$$\ell : E(G) \to \mathbb{N}, \quad uv \mapsto 1 + \rho(u) + \rho(v).$$

ρ should be chosen such that the induced vertex labeling

$$\sigma : V(G) \to \mathbb{N}_0, \quad v \mapsto \sum_{u \in N_G(v)} \ell(uv)$$

is a proper vertex coloring. Bensmail et al. conjecture that such a
$\rho : V(G) \to \{0, 1, \ldots, \Delta(G)\}$ exists for every graph G. We prove their conjecture for a few graph classes [2].

References

[1] J. Bensmail, C. Marcille, and M. Orenga, Pushing Vertices to Make Graphs Irregular, hal-04810955.

[2] D. Rautenbach, L. Schwartze, F. Werner, Coloring by Pushing Vertices, 2025, `https://arxiv.org/abs/2505.05252`.