Representing Distance-Hereditary Graphs with Trees

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Cographs are precisely the undirected graphs that can be represented by a vertex-labelled tree, that is, a pair (T, t), where T is a rooted tree and t is a labelling of the vertices of T into the set $\{0, 1\}$. Specifically, we say that the pair (T, t) explains G if T has leaf set $V(G)$ and, for any two distinct vertices x and y of G, x and y are joined by an edge in G if and only if the least common ancestor of x and y in T has label 1 via t.

Recently [1], the class of arboreal networks was introduced as a generalization of rooted trees. Arboreal networks are directed, acyclic graphs whose underlying, undirected graph is a tree. Intuitively, they are rooted trees which can have more than one root. This led to the question of characterizing those undirected graphs G that can be explained by a vertex-labelled arboreal network (N, t), in the same way cographs are explained by vertex-labelled trees. Interestingly, this is a well known and well studied class of graphs [2].

References

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[2] G. E. Scholz, Representing distance-hereditary graphs with multi-rooted trees, *Graphs and Combinatorics* 2025 (in press).