3-colourability, diamonds and butterflies

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The 3-colourability problem is an NP-complete problem which remains NP-complete for graphs with maximum degree four, for claw-free graphs, and even for (claw,diamond)-free graphs. In this talk we will consider induced subgraphs, among them are the *claw* ($K_{1,3}$), the *diamond* (the graph $K_4 - e$), the *butterfly* (two triangles sharing a vertex), and the generalized *net* $N_{i,j,k}$ (a triangle with three attached paths with i, j, k edges).

Our main result is a complete characterization of all 3-colourable (*claw, diamond, H*)-free graphs for $H \in \{N_{1,1,1}, N_{1,1,2}, N_{1,2,2}, N_{2,2,2}\}$. We will present a description of all non 3-colourable (*claw, diamond, H*)-free graphs for $H \in \{N_{1,1,1}, N_{1,1,2}, N_{1,2,2}, N_{2,2,2}\}$ in terms of butterflies. Moreover, we will show extensions of this characterization to larger graph classes.

References

[1] B. Randerath and I. Schiermeyer, *Vertex colouring and forbidden subgraphs - a survey*, **Graphs and Combinatorics** 20 (2004) 1-40.

[2] I. Schiermeyer and B. Randerath, *Polynomial χ-Binding Functions and Forbidden Induced Subgraphs: A Survey*, **Graphs and Combinatorics** 35 (1) (2019) 1–31.