Disjoint dominating and 2-dominating sets in graphs: Hardness and Approximation results

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A set $D \subseteq V$ of a graph $G = (V, E)$ is a dominating set of G if each vertex $v \in V \setminus D$ is adjacent to at least one vertex in D, whereas a set $D_2 \subseteq V$ is a 2-dominating (double dominating) set of G if each vertex $v \in V \setminus D_2$ is adjacent to at least two vertices in D_2. A graph G is a DD_2-graph if there exists a pair (D, D_2) of disjoint dominating set and 2-dominating set of G. In [1], several open problems related to DD_2-graphs were posed. In this paper, we answers some of these problems and present the following results: we provide an approximation algorithm for the problem of determining a minimal spanning DD_2-graph of minimum size (MIN-DD_2) with an approximation ratio of 3; a minimal spanning DD_2-graph of maximum size (MAX-DD_2) with an approximation ratio of 3; and the smallest number of edges which when added to a non-DD_2-graph results in a minimal spanning DD_2-graph for any graph (MIN-TO-DD_2) with an $O(\log n)$ approximation ratio. Additionally, we prove that MIN-DD_2 and MAX-DD_2 are APX-complete for graphs with maximum degree 4. Moreover, for a 3-regular graph, we show that MIN-DD_2 and MAX-DD_2 are approximable within a factor of 1.8 and 1.5 respectively.

References

[1] M.Miotk, J.Topp, and P.Żyliński. *Disjoint dominating and 2-dominating sets in graphs*, Discrete Optimization, 35:100553, (2020).