Zombies on the Grid

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In Zombies and Survivors, a set of zombies attempts to eat a lone survivor, Mindy, loose on a given connected graph. The zombies randomly choose their initial location, and during the course of the game, move directly toward Mindy. At each round, they move to the neighbouring vertex that minimizes the distance to Mindy; if there is more than one such vertex, then they choose one uniformly at random. Mindy attempts to escape from the zombies by moving to a neighbouring vertex or staying on her current vertex. The zombies win if eventually one of them eats Mindy by landing on her vertex; otherwise, Mindy wins. The zombie number $z(G)$ of a graph G is the minimum number of zombies needed to play such that the probability that they win is at least $1/2$.

In this paper, we investigate the game played on toroidal grids $C_n \Box C_n$. In particular, we show that asymptotically almost surely $z(C_n \Box C_n) = \Omega(n/\log^c n)$ for some constant c and that $z(C_n \Box C_n) = O(n^{3/2})$.