Perfect (1,2)-Dominating Sets in Graphs with a Few Large-Degree Vertices

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Let $k \geq 1$ be an integer. A subset $D \subseteq V(G)$ is a $(1,k)$-dominating set if for every vertex $v \in V(G) \setminus D$ there exist $u, w \in D$ such that $uv \in E(G)$ and $d_G(v,w) \leq k$. The concept of $(1,2)$-dominating sets was introduced in [1] and further studied in [2, 3], where A. Michalski defined a *proper* $(1,2)$*-dominating set* as a $(1,2)$-dominating set that is not $(1,1)$-dominating. Based on this idea, in [4] we introduced a *perfect* $(1,2)$*-dominating set* (shortly $(1,2)$-PDS) as a $(1,2)$-dominating set in which every vertex outside D is adjacent to exactly one vertex of D.

In this talk, we investigate the existence of $(1,2)$-PDS in graphs containing at most two vertices of maximum degree. In particular, we provide a complete characterization for the cases $\Delta(G) = n-1$ and $\Delta(G) = n-2$.

References

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