Constructions of Turán systems that are tight up to a multiplicative constant

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The Turán density $t(s,r)$ is the asymptotically smallest edge density of an r-graph for which every set of s vertices contains at least one edge. The question of estimating this function received a lot of attention since it was first raised by Turán in 1941. A trivial lower bound is $t(s,r) \geq 1/\binom{s}{s-r}$. In the early 1990s, de Caen [1] conjectured that $t(r+1,r)$ grows faster than $O(1/r)$ and offered 500 Canadian dollars for resolving this question.

I will give an overview of this area and present a construction from [2] disproving this conjecture by showing more strongly that for every integer R there is C such that $t(r+R,r) \leq C/\binom{r+R}{R}$, that is, the trivial lower bound is tight for every fixed R up to a multiplicative constant $C = C(R)$.

References

[1] D. de Caen. The current status of Turán's problem on hypergraphs. In *Extremal problems for finite sets (Visegrád, 1991)*, volume 3 of *Bolyai Soc. Math. Stud.*, pages 187–197. János Bolyai Math. Soc., Budapest, 1994.

[2] O. Pikhurko, *Constructions of Turán systems that are tight up to a multiplicative constant*, Advances in Mathematics **464** (2025), 110148, 11 pages.