Some recent results on modular product

Iztok Peterin[1,2]

[1] Faculty of Electical Engineering and Computer Science, University of Maribor
[2] Institute of Mathematics, Physics and Mechanics, Ljubljana

A product of two graphs G and H has a vertex set $V(G) \times V(H)$. For edges, we consider three objects in every factor for the definition of an edge in a product: either a vertex or an edge or a non-edge. Combining the mention objects from one factor with the same objects in the other factor gives eight different possibilities (notice that vertex by vertex must be ignored to avoid loops) to have an edge or a non-edge in a product. So, all together $2^8 = 256$ different graph products exists and four of them—Cartesian $G \square H$, strong $G \boxtimes H$, direct $G \times H$ and lexicographic $G \circ H$—gain a special status and are called standard products.

Among all graph products only 10 are associative and commutative. We join them into pairs of a product $G * H$ together with its complementary product $G \overline{*} H \overset{\text{def}}{=} \overline{\overline{G} * \overline{H}}$ where $\overline{G}$ is the complement of a graph G. Six of them represent either a standard product (Cartesian, strong or direct) or its complementary product, next two are empty and complete product and finally modular product $G \diamond H$, where $E(G \diamond H) = E(G \square H) \cup E(G \times H) \cup E(\overline{G} \times \overline{H})$, and its complementary product.

We present several recent results on distance [2], domination number [1] and independence sets of modular products with respect to some properties of their factors.

References

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