Cycle decompositions of circulants $C(n, \{1, 3\})$

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A *decomposition* of a graph G is a collection of edge-disjoint subgraphs $H_1, H_2, \ldots, H_t$ of G such that each edge of G belongs to exactly one H_i. We call this collection a *k-factorization* when every H_i is a k-regular spanning subgraph of G.

For a positive integer n and a set $S \subseteq \{1, \ldots, \lfloor(\frac{n}{2}\rfloor)\}$ a *circulant* $C(n, S)$ is a graph $G = (V, E)$ such that $V = \mathbb{Z}_n$ and $E = \{\{u, v\} : \delta(u, v) \in S\}$ where $\delta(u, v) = min\{\pm|u - v| \pmod n)\}$.

Some results on decomposition of those graphs into cycles were obtained. Inspired by the work of Bryant and Martin [1], who gave a complete solution for the cycle decomposition of $C(n, \{1, 2\})$, we examine the case when $S = \{1, 3\}$. Among others, we present the results on decomposition of $C(n, \{1, 3\})$ into cycles of odd lengths and into cycles of even lengths.

In [2] Bryant showed that, whenever $n \geq 5$, there exists a 2-factorization of $C(n, \{1, 2\})$ in which one factor is a Hamiltonian cycle and the other factor is isomorphic to any given 2-regular graph of order n. We discuss some open problems concerning the 2-factorization of $C(n, \{1, 3\})$.

References

[1] E.D.Bryant, M.Geoffrey, Some results on decompositions of low degree circulant graphs, *Australas. J Comb. 45* 2009 pp.251-262.

[2] D.Bryant, Hamilton cycle rich two-factorizations of complete graphs, *Journal of Combinatorial Designs 12.2* 2004 pp.147-155.