A generalization of an ear decomposition and k-trees in highly connected star-free graphs

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In this talk, we introduce a generalized version of an ear decomposition, called a j-spider decomposition, for j-connected star-free graphs with $j \geq 2$. Its application enables us to improve a previousely known sufficient condition for the existence of a k-tree in highly connected star-free graphs, where a k-tree is a spanning tree in which every vertex is of degree at most k. More precisely, we show that every j-connected $K_{1,j(k-2)+2}$-free graph has a k-tree for $k \geq j$, thereby improving a classical result of Jackson and Wormald [1] for $k \geq j$. Our approach differs from previous studies based on toughness-type arguments and instead relies on both a j-spider decomposition and a factor theorem related to Hall's marriage theorem.

This talk is based on the paper [2].

References

[1] B.Jackson and N.C.Wormald, k-walks of graphs, *Australas. J. Combin.* **2** (1990) pp. 135-146.

[2] S.Maezawa, K.Ozeki, M.Yamamoto, and T.Yashima, A generalization of an ear decomposition and k-trees in highly connected star-free graphs, arXiv:2508.05962.