Edge-uncoverability by four perfect matchings in cubic graphs

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Several longstanding conjectures in graph theory, including the Cycle Double Cover Conjecture, can be reduced to the case of cubic graphs. A notable parameter in this context is the perfect matching index of a cubic graph, defined as the minimum number of perfect matchings needed to cover its edges. In particular, if these conjectures hold for cubic graphs with perfect matching index at least 5, they hold in general.

In this talk, we introduce several invariants that capture how far a cubic graph G is from being coverable by four perfect matchings. One such invariant is the *four perfect matching defect* of G, denoted by $d_{PM}(G)$, defined as the minimum number of edges of G not covered by four perfect matchings. Another is the *four matching cover defect* of G, denoted by $d_M(G)$, defined as the minimum sum of defects of matchings over all matching covers of G containing four matchings, where the *defect of a matching* is half of the number of vertices it leaves uncovered. We prove that $d_M(G) \leq d_{PM}(G)$. Furthermore, we show that for each integer k, there exists a cubic graph with $d_M(G) = d_{PM}(G) = k$; that is, there are cubic graphs that are far from being coverable by four perfect matchings. We also present another family of cubic graphs, in which, for each integer k, there exists a cubic graph with $2d_M(G) \leq d_{PM}(G) = 2k$.

Beyond offering new perspectives on the structure of cubic graphs, we also demonstrate that such measures yield partial results toward resolving these famous conjectures.