Balanced generalized kite designs

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In this work, we study valuations and labelings of bipartite graphs and their applications to cyclic graph designs. In particular, we introduce the notion of (A, B)-ordered and uniformly ordered labelings for a bipartite graph $G = (V, E)$ with partition classes A and B. Using these labelings, we construct (A, B)-uniformly ordered labelings and describe how shifts of the labeling modulo $r + 1$ preserve certain ordering properties.

Our main result is a constructive method for cyclic $(C_m + P_{n+1})$-designs of order v, where $v \equiv 1 \mod 2(m + n)$. These results illustrate the interplay between ordered labelings of bipartite graphs and the construction of balanced generalized kite designs.

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