Paired versus double domination in forbidden graph classes

Michael A. Henning[(1)], Paras Vinubhai Maniya[(2)],
Dinabandhu Pradhan[(2)]

[(1)] University of Johannesburg, Auckland Park, South Africa
[(2)] Indian Institute of Technology (ISM), Dhanbad, India

A set D of vertices in a graph G is a dominating set of G if every vertex not in D has a neighbor in D, where two vertices are neighbors if they are adjacent. If the dominating set D of G has the additional property that the subgraph induced by D contains a perfect matching (not necessarily as an induced subgraph), then D is a paired dominating set of G. The paired domination number of G, denoted by $\gamma_{pr}(G)$, is the minimum cardinality of a paired dominating set of G. A set $D \subseteq V(G)$ is a double dominating set of G if every vertex in $V(G) \setminus D$ has at least two neighbors in D, and every vertex in D has a neighbor in D. The double domination number of G, denoted by $\gamma_{\times 2}(G)$, is the minimum cardinality of a double dominating set of G. Chellali and Haynes [2] showed that if G is a claw-free graph without isolated vertices, then the paired domination number of G is at most the double domination number of G. In this paper, we show that if G is a H-free graph for some $H \in \{P_5, 2K_2 \cup K_1, \text{fork}\}$ without isolated vertices, then $\gamma_{pr}(G) \leq \gamma_{\times 2}(G)$.

References

[1] M. Blidia, M. Chellali, T. W. Haynes, Characterizations of trees with equal paired and double domination numbers. *Discrete Math.* **306** (2006), 1840–1845.

[2] M. Chellali and T. W. Haynes, On paired and double domination in graphs. *Util. Math.* **67** (2005), 161–71.