Colouring cubic multipoles

R. Lukoťka[(1)]

[(1)] Comenius University, Bratislava, Slovakia

To study 3-edge-uncolourability of a cubic graph one can take a cut containing k edges and split the graph into two graph parts, called cubic k-poles. Each 3-edge-colouring of a k-pole induces a k-tuple of colours on the dangling edges, called boundary colouring. All colourings of the k-pole induce a (multi)set of boundary colourings, called colouring set. A colouring set contains only colourings satisfying parity lemma and the set has to be closed under Kempe switches. For $k \leq 5$ these two conditions are not only necessary but also sufficient. We will focus on the case where $k = 6$. We introduce a new equivalence relation that greatly reduces the number of colouring sets one needs to consider. We present the results of computational experiments using this equivalence relation.

For planar graphs Four Colour Theorem can be used to restrict colouring sets of k-poles. We show that certain generalised flow polynomials are an efficient tool to capture the number of colourings with given boundary. We explore conditions that Four Colour Theorem imposes on the polynomial and study k-poles that are close to "refuting" the Four Colour Theorem with respect to their polynomial coefficients.