Construction of k-matchings in graph products

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The study of classical graph invariants—such as chromatic, domination, and independence numbers—in graph products has received significant attention. Here, we focus on variations of matchings in the four standard graph products: Cartesian, strong, direct and lexicographic. Specifically, we define a subset $M \subseteq E$ of a graph $G = (V, E)$ as a k-*matching* if the edges in M induce a k-regular subgraph of G.

Summarizing results in [1], we present explicit constructions of k-matchings in graph products $G \star H$, utilizing k_G-matchings M_G and k_H-matchings M_H from the factor graphs G and H. Although these constructions do not always yield maximum k-matchings for the product, they achieve the largest possible size among all k-matchings that are weak-homomorphism preserving – meaning that matched edges in the product never project onto unmatched edges in the factors.

References

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