Random embeddings of bounded degree trees with optimal spread

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A seminal result of Komlós, Sárközy, and Szemerédi [1] states that any n-vertex graph G with minimum degree at least $(1/2 + \alpha)n$ contains every n-vertex tree T of bounded degree. Recently, Pham, Sah, Sawhney, and Simkin [2] extended this result to show that such graphs G in fact support an optimally spread distribution on copies of a given T, which implies, using the recent breakthroughs on the Kahn-Kalai conjecture, the robustness result that T is a subgraph of sparse random subgraphs of G as well. Pham, Sah, Sawhney, and Simkin construct their optimally spread distribution by following closely the original proof of the Komlós-Sárközy-Szemerédi theorem which uses the blow-up lemma and the Szemerédi regularity lemma. We give an alternative, regularity-free construction that instead uses the Komlós-Sárközy-Szemerédi theorem (which has a regularity-free proof due to Kathapurkar and Montgomery) as a black-box. Our proof is based on the simple and general insight that, if G has linear minimum degree, almost all constant sized subgraphs of G inherit the same minimum degree condition that G has.

References

[1] J. Komlós, G. Sárközy, and E. Szemerédi, Proof of a packing conjecture of Bollobás, *Combinatorics, Probability and Computing* vol.4 1995 pp. 241–255.

[2] H. Pham, A. Sah, M. Sawhney, and M. Simkin, A toolkit for robust thresholds, *arXiv:2210.03064 [math.CO]*, 2022.