Line graph orientations and list edge colorings of regular graphs

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The List Edge Coloring Conjecture states that for any graph G, the list chromatic index $ch'(G)$ equals the chromatic index $\chi'(G)$. A major breakthrough toward resolving this conjecture was Galvin's proof that it holds for bipartite graphs. It is natural to consider extending his coloring procedure to general graphs by decomposing them into bipartite subgraphs. However, such decompositions turn out to be incompatible with the method.

In 1996, Kahn proved that the conjecture holds asymptotically, establishing an upper bound of $\chi'(G) + o(\chi'(G))$. A later refinement by Häggkvist and Janssen, yielding a bound of $\chi'(G) + \widetilde{O}(\chi'(G)^{2/3})$, relies on the Alon–Tarsi polynomial method. This approach derives bounds from the existence of specific orientations of the line graph. Interestingly, such orientations can be constructed from those of bipartite subgraphs arising from natural decompositions. Therefore, any improvement of the result for bipartite graphs could potentially enhance the general bounds. Unfortunately, no such bipartite-specific improvements are currently known.

In this work, we explore this approach and present partial results on coloring line graphs of complete multipartite graphs.