Constructing algebraic expressions for lattice-structured digraphs

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We investigate relationship between algebraic expressions and *labeled two-terminal directed acyclic graphs* (labeled *st-dags*), in which each edge carries a unique label. An algebraic expression is referred to as an *st-dag expression* if it is algebraically equivalent to the sum of edge-label products over all spanning paths of the st-dag. The st-dags considered here are based on lattice structure and have $m \times n$ vertices (m rows and n columns). Examples are shown in Figure 1: (a) *grid graph* $G_{m,n}$, (b) *triangular grid* $T_{m,n}$, (c) *king graph* $K_{m,n}$. We treat m as a constant representing the graph's *depth*, while n determines its *size*. Our objective is to simplify the expressions for these graphs. To this end, we apply *backtracking* and *decomposition* methods (*BM* and *DM*, respectively) which generate expressions for these graphs, and we estimate the lengths of the generated expressions as functions of n. BM produces expressions of length $O\left(n^m\right)$ for both $G_{m,n}$ and $T_{m,n}$, and of exponential length in n for $K_{m,n}$. In contrast, DM yields more compact expressions: length $O\left(n \log^{m-1} n\right)$ for both $G_{m,n}$ and $T_{m,n}$ and length $O\left(n^{\log_2(4m-2)}\right)$ for $K_{m,n}$.

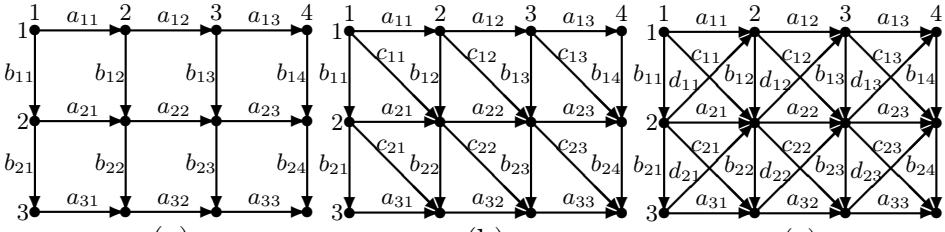

Figure 1: Lattice-structured digraphs.