How can graphs help us count?

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This talk is about the topic of *Boundary Matrices*, which are matrices specified by a set of forbidden submatrices that these Boundary Matrices must avoid containing. In this sense, these matrices are a generalisation of the antidictionary languages, which is a classical - solved - problem [1]. Ideas similar to boundary matrices show up in various areas of research [2], [3].

In addition to asking "how many boundary matrices without forbidden submatrices $Z_1, \ldots, Z_x$ are there of size $m \times n$?", our focus of the project is to examine the relation between the numbers of these matrices. The main interest is answering a conjecture from [4], whether there exists a two-dimensional linear recurrence relation between these numbers of boundary matrices.

As we will see, the adjacency matrix of a directed graph related to building these boundary matrices plays a key role in determining the answer to the conjecture.

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