Odd coloring of k-trees

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For a graph G, an odd coloring of G is a proper coloring φ such that every non-isolated vertex v has a color c such that $|\varphi^{-1}(c) \cap N(v)|$ is an odd integer. A graph is said to be odd k-colorable if it admits an odd coloring with at most k colors. This notion was introduced by Petruševski and Škrekovski [2] in 2022, where they investigated odd coloring of planar graphs.

In this talk, we focus on odd coloring of k-trees. For a positive integer k, a graph which is obtained from K_{k+1} by recursively adding a vertex which is joined to a clique of order k is called a k-tree. For any $k \geq 1$, it is easy to see that there are infinitely many k-trees that are not odd $(k+1)$-colorable. On the other hand, according to a result by Cranston et al. [1], it follows that every graph of tree-width at most k is odd $(2k+1)$-colorable, and hence every k-tree is odd $(2k+1)$-colorable. We improve this bound by showing that every k-tree is odd $(k + 2\lfloor \log_2 k \rfloor + 3)$-colorable. Furthermore, when $k = 2, 3$, we show that every 2-tree is odd 4-colorable and that every 3-tree is odd 5-colorable, both of which are tight bounds. In particular, since every maximal outerplanar graph is a 2-tree, this implies that every maximal outerplanar graph is odd 4-colorable.

References

[1] D. W. Cranston, M. Lafferty, Z.-X. Song, A note on odd colorings of 1-planar graphs, *Discrete Appl. Math.*, 2023 pp.112-117.

[2] M. Petruševski, R. Škrekovski, Colorings with neighborhood parity condition, *Discrete Appl. Math.* 2022 pp.385-391.