Coloring Mixed Graphs

Grzegorz Gutowski[(1)], Konstanty Junosza-Szaniawski[(2)], Antonio Lauerbach[(3)], Paweł Rzążewski[(2)], Alexander Wolff[(3)]

[(1)] Jagiellonian University
[(2)] Warsaw University of Technology
[(3)] University of Würzburg

A *mixed graph* is defined as a triple (V, E, A), where V is the set of vertices, E is the set of undirected edges, and A is the set of directed edges (arcs). A proper coloring of a mixed graph is an assignment of positive integers (colors) to the vertices such that adjacent vertices connected by an undirected edge receive distinct colors, and for every directed edge $(u, v) \in A$, the color assigned to vertex v is strictly greater than the color assigned to u.

In this talk, we present two algorithms for computing the minimal number of colors required to properly color a mixed graph. The first algorithm operates in exponential space, while the second in polynomial space. We further analyze the computational complexity of these algorithms, revealing connections to several interesting problems in extremal graph theory.