On relative simple Heffter spaces

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Let G be an abelian group and suppose that J is a subgroup of G of order t, a *half-set* V of $\mathrm{G}\setminus J$ is a subset of $\mathrm{G}\setminus J$ such that for each non-involution element $x \in \mathrm{G}\setminus J$, either x or $-x$ is contained in V and any involution elements of $\mathrm{G}\setminus J$ are also contained in V. An $(nk, k)_t$ *relative Heffter system* is a partition of a half-set V of $\mathrm{G}\setminus J$ into zero-sum blocks of equal size. Two $(nk, k)_t$ relative Heffter systems $\mathcal{P}$ and $\mathcal{Q}$, based on the same half-set, are said to be *orthogonal* if their blocks intersect in at most one element. In a [1], we introduce the concept of a $(nk, k; r)_t$ *relative Heffter space*, which is a collection of r mutually orthogonal Heffter systems. This definition naturally generalises the concepts of a relative Heffter array and a Heffter space.

The *density* of a Heffter space refers to the density of the collinear graph associated with the Heffter space. In the paper this talk is based on [1], we present two constructions of infinite families of relative Heffter spaces, that satisfy the additional property of being globally simple. One of these constructions always achieves maximal density. By obtaining results on globally simple Heffter spaces, [1] also obtains new results on mutually orthogonal cycle decompositions and biembeddings of cyclic cycle decompositions of the complete multipartite graph into an orientable surface.

References

[1] L. Johnson, L. Mella, A. Pasotti, On relative simple Heffter spaces, arXiv preprint arXiv:2503.07445, (2025).