Asymmetric depth of graphs

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A partial automorphism of a graph is an isomorphism between its induced subgraphs. The set of all partial automorphisms of a given graph forms an inverse monoid under composition of partial maps and taking partial inverses. In constrast to classical group theory approach to studying symmetries of graphs, where the automorphism group of a graph can be (and almost always is) trivial, the inverse monoid of partial automorphisms of a graph is never trivial. It gives full algebraic description of the graph [1]. In our talk, motivated by the study of partial automorphism inverse monoids of graphs, we investigate symmetries of graphs utilising the measure of asymmetric depth of graphs defined through the rank of the largest nontrivial partial automorphism. We establish a new, tight lower bound for the asymmetric depth of any simple graph Γ of order n. Any graph achieving this bound must be a strongly regular graph with parameters $(n, \frac{n-1}{2}, \frac{n-5}{4}, \frac{n-1}{4})$ also known as *conference graph*. Via computation performed on a high-performance cluster, we were able to identify an asymmetric conference graph on 37 vertices that meets this bound, thereby proving its tightness. We also show that it is one of the smallest possible graphs to meet this bound.

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References

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