Connections between local regularity and symmetry in graphs

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When talking about highly symmetric graphs, we may be talking about several different but interconnected concepts. We may be talking about graphs with large automorphism groups (in comparison to their orders), or graphs whose automorphism groups act in a special way (vertex-transitive, edge-transitive, arc-transitive, etc.), or we can even be talking about graphs that appear symmetric (but may not necessarily be), by which we may mean some kind of a regularity shared by graphs from the other two classes defined by symmetries viewed as automorphisms.

In our presentation, we aim to connect these various concepts of symmetry via discussing hierarchies within the above classes of graphs and the links connecting the various levels of these hierarchies. We will also discuss possibilities for loosening or generalizing some of the discussed concepts in order to explore connections to even wider classes defined via the existence or possibly even lack of specific symmetries. Some of the topics discussed will include smallest vertex-transitive subgroups and families of automorphisms and their connection to Cayley and quasi-Cayley graphs, the monoid of partial automorphisms of a graph, and girth-regular, edge-girth-regular, and vertex-girth-regular graphs.

Parts of this talk may also be viewed as an introduction to presentations included in the *Algebraic Graph Theory* session of the 10th Cracow Conference 2025.