Hitting times and the power of choice for random geometric graphs

D. Ignasiak[(1)], L. Lichev[(1)]

[(1)] Technical University of Vienna, Vienna, Austria

We consider a random geometric graph process where random points $(X_i)_{i\geq 1}$ are embedded consecutively in the d-dimensional unit torus $\mathbb{T}^d$, and every two points at distance at most r form an edge. As $r \to 0$, we confirm that well-known hitting time results for k-connectivity (with $k \geq 1$ fixed) and Hamiltonicity in the Erdős-Rényi graph process also hold for the considered geometric analogue. Moreover, we exhibit a sort of probabilistic monotonicity for each of these properties.

We also study a geometric analogue of the power of choice where, at each step, an agent is given two random points sampled independently and uniformly from $\mathbb{T}^d$ and has to add exactly one of them to the already constructed point set. When the agent is allowed to make their choice with the knowledge of the entire sequence of random points (offline 2-choice), we show that they can construct a connected graph at the first time t when none of the first t pairs of proposed points contains two isolated vertices in the graph induced by $(X_i)_{i=1}^{2t}$, and maintain connectivity thereafter. We also derive analogous results for k-connectivity and Hamiltonicity. This shows that each of the said properties can be attained two times faster (time-wise) and with four times fewer points in the offline 2-choice process compared to the 1-choice process.

In the online version where the agent only knows the process until the current time step, we show that k-connectivity and Hamiltonicity cannot be significantly accelerated (time-wise) but may be realised on two times fewer points compared to the 1-choice analogue.