K-Coloring $(bull, chair)$-free graphs

N. Hodur[(1)], M. Pilśniak[(1)], M. Prorok[(1)], P. Rzążewski[(2)]

[(1)] AGH University of Krakow, Poland
[(2)] Warsaw University of Technology & University of Warsaw, Poland

The k-Coloring problem is NP-hard in general, but it becomes tractable in some hereditary graph classes. We show that it can be solved in polynomial time for $(bull, chair)$-free graphs. Here, *chair* is a 3-star $S_{1,1,2}$ with one edge subdivided and bull is a triangle with two additional leaves attached to two vertices.

The algorithm we present in this talk resolves even a more general List k-Coloring problem: given a graph G and a set of lists $\{L(v) : v \in V(G), L(v) \subset [k]\}$, we look for a proper coloring c of $V(G)$ such that $c(v) \in L(v)$ for every vertex v. The algorithm works recursively, where base trivial case is $|V(G)| = 1$ or $\max_{v \in V(G)} |L(v)| = 1$. In one step we exhaustively guess the coloring of an expansion of path $R \subset G$ and for each coloring guessed we adjust the lists and call the algorithm on the components of $G - R$. In each descendant call maximum length of the lists decreases, so the depth of recursion is bounded by k.

References

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