Domination type parameters in 3-regular and 4-regular graphs

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A set S of vertices in a graph G is a dominating set if every vertex in $V(G) \setminus S$ is adjacent to a vertex in S. The domination number, $\gamma(G)$, of G is the minimum cardinality among all dominating set in G. We discuss best possible upper bounds on domination-type parameters in cubic graphs. Among other results, we show that if G is a cubic graph of order n, then $\gamma_{t2}(G) \leq \frac{2}{5}n$ and $\gamma_r(G) \leq \frac{2}{5}n$, where $\gamma_{t2}(G)$ and $\gamma_r(G)$ denote the semitotal and restrained domination numbers, respectively. The $\frac{1}{3}$-conjecture for domination in 4-regular graphs states that if G is a 4-regular graph of order n, then $\gamma(G) \leq \frac{1}{3}n$. We prove this conjecture when G has no induced 4-cycle. A thorough treatise on dominating sets can be found in [1, 2, 3].

References

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