Extremal problems on planar graphs

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Let $\mathrm{ex}_{\mathcal{P}}(n, T, H)$ denote the maximum number of copies of T in an n-vertex planar graph which does not contain H as a subgraph. When $T = K_2$, $\mathrm{ex}_{\mathcal{P}}(n, T, H)$ is the planar Turán number of H, denoted by $\mathrm{ex}_{\mathcal{P}}(n, H)$. The topic of extremal planar graphs was initiated by Dowden (2016) [1]. He obtained sharp upper bound for both $\mathrm{ex}_{\mathcal{P}}(n, C_4)$ and $\mathrm{ex}_{\mathcal{P}}(n, C_5)$. In [2], we gave a sharp upper bound $\mathrm{ex}_{\mathcal{P}}(n, C_6) \leq \frac{5}{2}n - 7$, for all $n \geq 18$. We also pose a conjecture on $\mathrm{ex}_{\mathcal{P}}(n, C_k)$, for $k \geq 7$.

We [3] proved that for every integer $n \geq 6$, $\mathrm{ex}_{\mathcal{P}}(n, C_5, \emptyset)$ is $2n^2 - 10n + 12 + \mathbb{1}_{n=7}$.

And (see [4]) for every fixed $k \geq 3$, $\mathrm{ex}_{\mathcal{P}}(n, C_{2k}, \emptyset)$ is $n^k/k^k + o(n^k)$. In this lecture, we present more recent similar results related to cycles and paths.

References

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