Complexity of Defensive Domination

Steven Chaplick[(1)], Grzegorz Gutowski[(2)], Tomasz Krawczyk[(3)]

[(1)] Maastricht University
[(2)] Jagiellonian University
[(3)] Warsaw University of Technology

In a graph G, a *k-attack* A is any set of at most k vertices and *ℓ-defense* D is a set of at most ℓ vertices. We say that defense D *counters* attack A if each $a \in A$ can be matched to a distinct *defender* $d \in D$ with a equal to d or a adjacent to d in G. In the *defensive domination problem*, we are interested in deciding, for a graph G and positive integers k and ℓ given on input, if there exists an ℓ-defense that counters every possible k-attack on G. Defensive domination is a natural resource allocation problem and can be used to model network robustness and security, disaster response strategies, and redundancy designs.

The defensive domination problem is naturally in the complexity class Σ_2^{P}. The problem was known to be NP-hard in general, and polynomial-time algorithms were found for some restricted graph classes. In this note, we prove that the defensive domination problem is Σ_2^{P}-complete.

We also introduce a natural variant of the defensive domination problem in which the defense is allowed to be a multiset of vertices. This variant is also Σ_2^{P}-complete, but we show that it admits a polynomial-time algorithm in the class of interval graphs. A similar result was known for the original setting in the class of proper interval graphs.

References

[1] Steven Chaplick, Grzegorz Gutowski, and Tomasz Krawczyk. A Note on the Complexity of Defensive Domination, 2025. `https://arxiv.org/abs/2504.14390`.