Weak and strong local irregularity of digraphs

I. Grzelec[1], A. Onderko[2], M. Woźniak[1]

[1] AGH University of Krakow, Kraków, Poland
[2] P.J. Šafárik University, Košice, Slovakia

Local Irregularity Conjecture states that every connected graph, except special cacti, can be decomposed into at most three *locally irregular graphs*, i.e., graphs in which adjacent vertices have different degrees [1, 2]. The notion of local irregularity was defined for digraphs in several different ways. At the beginning of this talk we present the already known concepts of local irregularity for digraphs with motivations, main conjectures and known results. Then we introduce the following new methods of defining a *locally irregular digraph*. The first one, *weak local irregularity*, is based on distinguishing adjacent vertices by *indegree-outdegree pairs*, and the second one, *strong local irregularity*, asks for *different balanced degrees* (i.e., difference between the outdegree and the indegree of a vertex) of adjacent vertices. For both of these irregularities, we define locally irregular decompositions and colorings of digraphs. We also provide related conjectures on the minimum number of colors in weak and strong locally irregular colorings and support them with new results for various classes of digraphs.

References

[1] O. Baudon, J. Bensmail, J. Przybyło, M. Woźniak, On decomposing regular graphs into locally irregular subgraphs, *Europ. J. Combin.*, **49** (2015), 90–104.

[2] J. Sedlar, R. Škrekovski, Local Irregularity Conjecture vs. cacti, *Discrete Appl. Math.* **343** (2024), 115–133.