Cubic girth-regular graphs of girth six

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Recall that given a graph Γ, the girth of Γ is the length of a shortest cycle in Γ. If the girth is finite, then each vertex v of Γ can be associated with a list of integers, one integer for each edge incident with v, representing the number of girth cycles containing the edge (possibly 0). If the lists associated with the vertices of Γ are all identical (and so Γ is regular), we say that Γ is *girth-regular*, and the shared list is said to be the *signature of* Γ. Note that all vertex-transitive graphs are necessarily girth-regular.

The concept of girth-regularity was introduced in [1], where several necessary conditions on signatures of cubic girth-regular graphs were proved, together with a classification of all cubic girth-regular graphs of girth at most 5. Consequently, all cubic vertex-transitive graphs of girth 6 were classified in [2]. In our work, we extend the latter to a characterization of all cubic girth-regular graphs of girth 6. In addition, we prove multiple additional conditions on signatures of cubic girth-regular graphs of any even girth.

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References

[1] P.Potočnik, J.Vidali, Girth-regular graphs, *Ars Math. Contemp.* 2019 pp.349–368.

[2] P.Potočnik, J.Vidali, Cubic vertex-transitive graphs of girth six, *Discrete Math.* 2022.