Largest planar graphs of diameter 3 and fixed maximum degree

Antoine Dailly[(1)], Sasha Darmon[(2)], Ugo Giocanti[(3)], Claire Hilaire[(4)], Petru Valicov[(5)]

[(1)] Université Clermont Auvergne, INRAE, France
[(2)] LBBE, Université Lyon 1, & INRIA, Université Lyon 1, France
[(3)] Faculty of Mathematics and Computer Science, Jagiellonian University, Kraków, Poland
[(4)] FAMNIT, University of Primorska, Koper, Slovenia
[(5)] LIRMM, Université de Montpellier, CNRS, Montpellier, France

The degree diameter problem asks for the maximum possible number of vertices in a graph of maximum degree Δ and diameter D. In this paper, we focus on planar graphs of diameter 3. Fellows, Hell and Seyffarth [1] proved that for all $\Delta \geq 8$, the maximum number $\mathrm{np}_{\Delta,D}$ of vertices of a planar graph with maximum degree at most Δ and diameter at most 3 satisfies $\frac{9}{2}\Delta - 3 \leq \mathrm{np}_{\Delta,3} \leq 8\Delta + 12$. We show that the given lower bound is tight up to an additive constant, by proving that there exists a constant $c > 0$ such that $\mathrm{np}_{\Delta,3} \leq \frac{9}{2}\Delta + c$ for every $\Delta \geq 0$. Our proof consists in a reduction to the fractional maximum matching problem on a specific class of planar graphs, for which we show that the optimal solution is $\frac{9}{2}$, and characterize all graphs attaining this bound.

References

[1] Michael R. Fellows, Pavol Hell, Karen Seyffarth Large planar graphs with given diameter and maximum degree, *Discrete Applied Mathematics* 1995 61(2) pp.133-153.