Odd Coloring: Complexity and Algorithms

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An odd k-coloring of a graph $G = (V, E)$ is a proper k-coloring of G such that for every non-isolated vertex $v \in V$, there exists at least one color that appears an odd number of times in the open neighborhood of v. The minimum k for which G admits an odd k-coloring is called the odd chromatic number of G and is denoted by $\chi_o(G)$. Given a graph G and a positive integer k, Decide Odd Coloring Problem is to decide whether G admits an odd k-coloring. Decide Odd Coloring Problem is known to be NP-complete for general graphs [1]. In this paper, we strengthen this hardness result by proving that Decide Odd Coloring Problem remains NP complete for dually chordal graphs. On the positive side, we prove that for any proper interval graph G, the odd chromatic number satisfies $\omega(G) \leq \chi_o(G) \leq \omega(G) + 1$. We further characterize the proper interval graphs for which $\chi_o(G) = \omega(G)$, and those for which $\chi_o(G) = \omega(G) + 1$. We present a linear-time algorithm to compute the odd chromatic number of block graphs. Finally, we prove that the odd chromatic number of an interval graph G is either $\omega(G)$ or $\omega(G)+1$. Further, we characterize the interval graphs having $\chi_o(G) = \omega(G)$ and $\chi_o(G) = \omega(G) + 1$.

References

[1] Yair Caro, Mirko Petruševski, Riste Škrekovski, Remarks on Odd Coloring of Graphs, *Discrete Applied Mathematics*, 321: 392-401, 2022.