Packing coloring of graphs with long paths

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A *packing coloring* of a graph G is a mapping $c : V(G) \to \mathbb{N}$ such that any two distinct vertices assigned color i are at distance greater than i in G. This generalizes classical proper coloring by incorporating distance constraints that grow with the color index. The smallest integer k for which such a coloring exists using colors $1, \ldots, k$ is called the *packing chromatic number*, denoted $\chi_p(G)$.

We define a new class of graphs called *path-aligned graph products*, denoted by $P_n \Diamond_l G$. Let n and l be positive integers such that $l \mid n$, and let G be a connected vertex-transitive graph that contains a path P_l as a subgraph.The graph $P_n \Diamond_l G$ is constructed as follows.

- Start with the path P_n, with vertex sequence $v_1, v_2, \ldots, v_n$.
- Partition P_n into n/l consecutive, disjoint subpaths of length l, i.e., the i-th subpath is $P_l^{(i)} = (v_{(i-1)l+1}, \ldots, v_{il})$ for $i = 1, 2, \ldots, n/l$.
- For each i, take a copy $G^{(i)}$ of the graph G, and identify the subpath $P_l^{(i)} \subseteq P_n$ with a fixed copy of $P_l \subseteq G^{(i)}$. That is, the vertices of $P_l^{(i)}$ are merged with the corresponding vertices of the embedded path P_l in $G^{(i)}$.

We investigate the packing chromatic number χ_p of such constructions for various choices of G, including cycles and complete graphs, and determine exact values or bounds in these cases. Furthermore, we extend our results to selected classes of corona products, including generalized coronas, which share similar alignment properties.