G-designs for some graphs on seven edges

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A *G-design of order n* is a collections of s edge disjoint graphs G_i isomorphic to G, whose union forms the complete graph K_n.

We complement a recent result by Fronček and Kubesa [2] by examining the remaining three disconnected bipartite graphs with seven edges: on nine and ten vertices. While the result itself is not too exciting, it provides an opportunity to present several different methods for finding G-designs. For $n \equiv 0, 1 \pmod{14}$, we use classical labeling methods introduced by Rosa [4] and generalized by El-Zanati et al. [3] and Bunge [1].

For $n \equiv 7 \pmod{14}$, we first decompose K_{14k+7} into $K_{7,7}$ and $K_{14} - K_7$ and then each of them to G. For $n \equiv 8 \pmod{14}$, we decompose K_{14k+8} into the circulant $Cir(14k+8; 1, 2, 7k+3, 7k+4)$ and its complement and then use labelings for the complement and labelings with some adjustments for the circulant.

References

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