Graham's rearrangement for a class of semidirect products

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A famous conjecture of Graham stated in 1971 asserts that for any set $A \subseteq \mathbb{Z}_p \setminus \{0\}$ there is an ordering $a_1, \ldots, a_{|A|}$ of the elements of A such that the partial sums $a_1, a_1+a_2, \ldots, a_1+a_2+\ldots+a_{|A|}$ are all distinct. Before the recently improvements, the state of the art was essentially that the conjecture holds when $|A| \leq 12$ and when A is a non-zero sum set of size $p-1$, $p-2$ or $p-3$. Many of the arguments for small A use the Polynomial Method and rely on Alon's Combinatorial Nullstellensatz. Very recently, Kravitz in [3], using a rectification argument, made a significant progress proving that the conjecture holds whenever $|A| \leq \log p/\log\log p$. A subsequent paper of Bedert and Kravitz [1] improved the logarithmic bound into a super-logarithmic one that is of the form $e^{c(\log p)^{1/4}}$ for some small constant $c > 0$.

In [2], we use a similar procedure to obtain an upper bound of the same type in the case of semidirect products $\mathbb{Z}_p \rtimes_\varphi H$ where $\varphi : H \to Aut(\mathbb{Z}_p)$ satisfies $\varphi(h) \in \{id, -id\}$ for each $h \in H$ and where H is abelian and each subset of H can be ordered such that all of its partial products are distinct.

References

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[3] N. Kravitz. Rearranging small sets for distinct partial sums, *Integers*, 24 (2024).