2-Domination edge subdivision in trees

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A set S of vertices in a graph G is a 2-dominating set of G if every vertex not in S has at least two neighbors in S, where two vertices are neighbors if they are adjacent. The 2-domination number of G, denoted by $\gamma_2(G)$, is the minimum cardinality among all 2-dominating sets in G. A γ_2-set of G is a 2-dominating set of G of cardinality $\gamma_2(G)$. The 2-domination subdivision number of G, denoted by $\mathrm{sd}_2(G)$, is the minimum number of edges which must be subdivided in order to increase the 2-domination number. If T is a tree of order $n \geq 3$, then $\mathrm{sd}_2(T) \leq 2$. In [1] we show that $\mathrm{sd}_2(T) = 1$ if and only if the set of vertices that belong to no γ_2-set of G is nonempty. A graph G is γ_2-q-critical if q is the least number such that for every subset of edges S of cardinality q, the graph produced by subdivision of S has a greater 2-domination number. If T is a γ_2-q-critical tree of order $n \geq 3$, then we prove that $q \leq n-2$. Among other results, we characterize γ_2-q-critical trees when q is large, namely $q \in \{n-4, n-3, n-2\}$. We also characterize γ_2-1-critical trees [1] and γ_2-2-critical trees [2].

References

[1] M.Dettlaff, M.A.Henning, M.Lemańska, A.Roux, J.Topp, 2-Domination edge subdivision in trees, *Australas. J. Combin.* to appear.

[2] M.Dettlaff, M.A.Henning, M.Lemańska, A.Roux, 2-Domination critical trees upon edge subdivision, *Australas. J. Combin.* 92(3) (2025), 357—381.