Further progress on Wojda's conjecture

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Two digraphs of order n are said to pack if they can be found as edge-disjoint subgraphs of the complete digraph of order n. It is well established that if the sum of the sizes of the two digraphs is at most $2n-2$, then they pack, with this bound being sharp. However, it is sufficient for the size of the smaller digraph to be only slightly below n for the sum of their sizes to significantly exceed this threshold while still guaranteeing the existence of a packing.

In 1985, Wojda conjectured that for any $2 \leq m \leq n/2$, if one digraph has size at most $n-m$ and the other has size less than $2n - \lfloor n/m \rfloor$, then the two digraphs pack. It was previously known that this conjecture holds for $m = \Omega(\sqrt{n})$. We confirm it for $m \geq 26$.

References

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